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U. S. Department of Agriculture,
WEATHER BUREAU.

# RAINFALL LAWS.

DEDUCED FROM

# TWENTY YEARS OF OBSERVATION.

BY

Dr GUSTAVUS HINRICHS.

Published by authority of the Secretary of Agriculture.

WASHINGTON, D. C.:
WEATHER BUREAU.
1893.

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# LETTER OF TRANSMITTAL.

U. S. DEPARTMENT OF AGRICULTURE,
WEATHER BUREAU,
*Washington, D. C., August* 15, 1893.

SIR: I have the honor to transmit herewith a paper entitled "Rainfall Laws, Deduced from Twenty Years of Observation," which has been submitted at my request by Dr. Gustavus Hinrichs, of St. Louis, Mo., and respectfully recommend that the same be printed in a limited edition in order to make available to meteorologists the rainfall investigations of this well-known scientist. A careful review of the paper suggests the advisability of printing it substantially as presented, though in form, as will be observed, it differs in some respects from previous publications of this Bureau.

Very respectfully,

MARK W. HARRINGTON,
*Chief of Weather Bureau.*

HON. J. STERLING MORTON,
*Secretary of Agriculture.*

# LETTER OF SUBMITTAL.

SAINT LOUIS, MO., *June* 30, 1892.

SIR: I have the honor to submit herewith the most general results of my investigations into the fundamental laws of rainfall, with especial reference to the rains useful and damaging to agriculture. This investigation of my own long-continued series of observations is expected to furnish a rational basis for all rainfall reductions.

Very respectfully,

GUSTAVUS HINRICHS.

MARK W. HARRINGTON,
*Chief of Weather Bureau.*

## TABLE OF CONTENTS.

# LIST OF ILLUSTRATIONS.

# RAINFALL LAWS, DEDUCED FROM TWENTY YEARS OF OBSERVATION.

## INTRODUCTORY REMARKS.

1. Sunshine and rainfall are the great powers that control all life on this globe. When they follow one another in proper order and intensity, crops grow luxuriously and ripen to harvests of abundance, and plenty quickens all the pulses of the nation. But if the order of succession or the distribution of intensity be greatly disturbed, the crops suffer in spite of the labors of the farmer, and a scanty harvest spreads stringency and want in every direction. If the disturbance be excessive, even famine may result, as we see to-day in the Valley of the Volga.

2. The condition of the air produced by these two powers we call the weather, and the science treating thereof is meteorology. The main object of this difficult science is the discovery of the laws manifest in the action of sunshine and rainfall.

3. The discovery of every new law concerning these forces endows us with new powers over nature. The great weather bureaus of the present day are but the beginning in this direction, the growth of barely a quarter of a century. The laws applied to-day limit the power of prediction essentially to a single day. This is, no doubt, a grand achievement when we look backwards, but it is only a small part of the possibility of science when we look forward in directions where seem to be marked the conditions that determine the general character of the coming season of a large region as definitely as we now see the weather of the coming day at our weather bureaus.

4. We also feel the necessity of the discovery of laws that will allow us to exert a direct influence over the coming weather. It is generally believed that the extended cultivation of the soil, and especially the planting or destroying of timber, has a definite influence on the climate. If so, man will be able to favorably modify climate in time.

5. Unfortunately authorities disagree on the law involved, some contending that the extent of forest is without influence on the rainfall while others see in the rapid devastation of our forests a sure sign of the deterioration of our climate.

6. The question loses none of its importance simply because it is an open one. It invites to more searching investigation, especially into the laws of rainfall.

7. Now, thus far, scientists have expressed the rainfall by two quantities, namely, the frequency and amount. The "rain frequency" is the number of days in any given period on which at least one hundredth of an inch of rain fell. The "amount" of rain is the total depth of water on a level surface, supposing that all loss by evaporation or other causes be prevented during the entire period considered.

8. Manifestly the same frequency and amount of rainfall may be produced by an almost infinite variety of actual rains. But it is the character and succession of these showers that determine whether the resulting effects are beneficent or not.

9. Let us suppose that a long continued series of observations have shown that a certain place has a yearly rainfall of 40 inches, and a frequency of 100; that is, it rains one hundred days in the year, and the total amount of water falling during the year would cover the level ground to the depth of 40 inches if all evaporation and loss were avoided.

10. Such places we have in the middle of our grand Mississippi Valley. With the distribution of the rains there prevailing, it produces an abundance almost every year; the climate is one of the most favored of the entire globe. A large portion of this valley has been brought into cultivation during the last half century.

11. But suppose that by any cause of nature or by the continued action of man in devastating forests and otherwise, the character of the rains should gradually change; suppose that heavy rains become more frequent, though the total frequency and amount remain exactly the same—100 and 40—would the climate remain favorable to the production of abundant crops?

12. Let us take an extreme case to make the answer to this important question decisive and self-evident. Suppose that on eleven days the rains measured 2 inches each, on ten days 1 inch, on seventy-eight days 0.10 inch, and one day 0.20 inch.

13. The eleven 2-inch rains would yield 22 inches, the ten 1-inch rains 10 inches, the seventy-eight 0.10-inch rains aggregrate 7.8 inches, total ninety-nine rains amounting to 39.8 inches, which by the 0.20-inch rain brings up the total for the year to 40 inches in amount and one hundred in number or frequency.

14. Now, such a rainfall would be that of a desert. The twenty-one heavy rains would wash the soil from the fields and flood everything, and the numerous light sprinkles would immediately evaporate without benefit to any useful plant.

15. Suppose we were in the possession of the results of a series of observations of rainfall for such a station in our Mississippi Valley, reaching back three centuries. Suppose the results of these observations showed that the average amount and frequency of the rains had remained unchanged; would that authorize us to say that the

settlement of the great valley had not changed the climate in respect to rainfall? Certainly not.

16. Before we could venture to express a reliable conclusion, we would have to go beyond the mere yearly summary of amount and frequency and study the character of the individual showers. If the original records were preserved, we would be able to ascertain whether the *distribution of the rains*, and accordingly the climate, had changed or not.

17. This shows clearly that the subject of rainfall calls for a much more profound study than it has received thus far. There is another, not less urgent, necessity for such study, in the requirements of agriculture.

18. Carefully watching the growth and ripening of my crops, and at the same time tabulating and summarizing my rainfall measurements, I discovered to my deep chagrin that the results did not march in parallel lines. The thrashing machine seemed to be entirely independent of my rain gauge.

19. Now, since both the thrashing machine and the rain gauge were correct, the error could only be in the statement of the results obtained by my rain gauge.

20. This form of statement, in strict accordance with the rules of the International Congress of Meteorologists, gave amount and frequency; consequently the thrashing machine has demonstrated this statement to be insufficient and misleading.

21. Accordingly, in the interest of rational agriculture, it is necessary to take up the entire question of rainfall and try to go to the bottom of it, independent of the present rules and regulations and however high the authorities that have established or deduced them.

22. There is still another reason for such study; it is felt by every man who desires to compare the climate of one place with that of another. Suppose, on account of intended travel or immigration, it is desired to compare the rainfall of Saxony and Iowa; say that of Chemnitz, Saxony, and Iowa City, Iowa.

23. For 1890 the Saxon Report (pp. 114–116) shows a total of 682.7 millimeters (equal to 26.89 inches) of rain on two hundred and fifty-one days; for Iowa City my observations gave 27.05 inches on ninety-seven days. The amount of rainfall was accordingly practically identical, but while at Chemnitz rain fell on two days out of every three, at Iowa City rain fell on only one day in four.

24. But upon further examination we find that at Chemnitz rain had been recorded on ninety-nine days which at Iowa City would not have been counted, being less in amount than the international limit adopted of 0.01 inch. These Chemnitz rain days, equal in number to all the rains counted at Iowa City, yielded only 3.5 millimeters, or 0.14 inch in the entire year. They were accordingly absolutely insig-

nificant, averaging but about 0.001 inch each. These rains ranged from a few drops, barely visible, to the lightest sort of a short sprinkle.

25. Leaving these ninety-nine merely nominal rain days out of account, there remain one hundred and fifty-two rain days at Chemnitz against ninety-seven at Iowa City in 1890; the rain frequency was as 3 to 2 at these two places.

26. In order to carry the comparison further we would soon be compelled to enter into a great many perplexing details. Yet, to draw lines of equal rain frequency on a map of a State or Territory, it would really be necessary to have a simple method of expressing this very complex mass of data.

27. Such a method I have obtained by the careful study of rainfall, to which the more important problems previously stated directed my attention many years ago.

28. I shall not now state the general laws of rainfall so discovered, nor give a brief indication of the applications which can be made thereof. I prefer to deduce and expose each of the general laws in the simplest and most direct manner in a *first part* of this investigation.

For a second part I reserve the exposition of the laws governing the normal distribution of rainfall throughout the seasons.

In a third part I shall give some of the laws controlling the abnormal distribution of rainfall, the extremes of which mark our floods and droughts. These laws will be useful for seasonal probabilities for the benefit of agriculture.

Finally, in a fourth part, the entire body of observations and reductions ought to be published in extenso.

The present publication comprises the first part of my research, which has been completed during the past six months.

## I.—RAINFALL INTENSITY.

29. The first condition of success in any quantitative research is the adoption of a suitable unit and scale of measure, so that the numbers used represent perfectly definite quantities.

30. Rainfall investigations thus far have not satisfied this first condition. It is therefore hardly to be wondered that so little of general importance has been brought to light by the very large bulk of work done on this subject.

31. Since modern meteorologists evidently consider rain frequency and amount suitable expressions for quantitative research, it is necessary to point out in detail that our statement is well founded in fact. A very few instances will suffice.

32. Turning to the table on page 94 it will be seen that the rainfall at Iowa City in 1889 amounted to 28.52 inches on ninety days,

while in 1890 an amount of 27.05 inches fell on ninety-seven days. Expressed in the units adopted by our International Congresses of Meteorologists, the rainfall of the two years named was practically identical at Iowa City.

33. But every farmer in the lower valley of the Iowa River knows that this scientific statement is absolutely contrary to fact. Such conflict between published scientific data and manifest practical experience hinders both the progress of science and the development of its application in the arts.

34. Now, let us consider the principal features of the rainfall of these two years by using the more complete data of our table.

35. We notice that in 1889 there were seven rain days with over 1 inch and two with over 2 inches of rainfall; in all, nine rain days with excessive rains, aggregating 13.02 inches. Of the total rainfall of 28.52 inches only 15.50 fell in moderate rains, almost the only ones that can benefit the growing crops.

36. In the year 1890 only four rain days exceeded 1 inch of rain, aggregating 5.88 inches in the entire year. Consequently, of the total rainfall of 27.05 inches, only 5.88 inches came in washing and flooding rains, and 21.17 inches in moderate showers, beneficial to the farmer.

37. To be quite exact, we ought yet to separate from these beneficial rains those that were insignificant. In 1889 there were thirty-nine days aggregating only 1.42 inches, while in 1890 there were thirty-five days aggregating 1.15 inches of rainfall.

38. The following table represents this practical analysis of the rainfall of the two years in as clear a light as possible:

| | 1889. | 1890. |
|---|---|---|
| | *Inches.* | *Inches.* |
| Total rainfall | 28.52 | 27.05 |
| Washing and flooding rains | 13.02 | 5.88 |
| Insignificant rains | 1.42 | 1.15 |
| Total useless or damaging rains | 14.44 | 7.03 |
| Total utilizable rains | 14.08 | 20.02 |

39. Now, my twenty year's normal values of total rainfall is 36 inches, of which about 16 inches came in ten heavy, damaging showers, and forty-three, amounting to 1.50 inches, in rains useless because of their insignificance, leaving 18.05 inches of moderate rains, which, so far as they come at the right season, are almost the only ones that can benefit the growing crops.

40. It thus appears that the useful rainfall of 1890 was in excess of normal, while that of 1889 was deficient by 22 per cent of the normal value. The useful rains of 1889 fell 30 per cent below those of 1890.

41. This simple matter-of-fact analysis of the actual measures obtained with the rain gauge agrees excellently with the summing up

of the results made quietly by the growing crops and noticed by the observing farmer, and finally proclaimed by the measurer at the thrashing machine.

42. It is true that the total rainfall amounted to the same number of inches in 1889 and 1890, and was low, being but little above two-thirds of the normal amount. But to the farmer the rainfall of the two years differed exceedingly; in 1890 the useful showers exceeded the normal value by 11 per cent, while in 1889 they fell 22 per cent below.

43. There can thus be no question about the correctness of the conclusion that meteorologists must break with the international method of expressing the rainfall results if they desire to present the observed facts in a manner that can benefit the farmer.

44. But this step is equally imperative in the interest of the science of meteorology itself. It may be advisable to show this by a couple of striking instances, such as may be taken from almost any one of the pages of our tables.

45. Turning to the table which for 1875 gives the rainfall by decades, we notice the rain frequency to be constant during the last five decades of summer. We see that from the second decade of July to the last decade of August, inclusive, there were three rain days in each of the five consecutive decades specified.

*Frequency and amount of rainfall, Iowa City, Iowa.*

| Year 1875. | Total. | | Rainfall intensity, *h*. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1. | | 10. | | 25. | | 50. | | 100. | | 200. | |
| | *f* | *a* | *f* | *a* | *f* | *a* | *f* | *a* | *f* | *a* | *f* | *a* | *f* | *a* |
| *July.* | | | | | | | | | | | | | | |
| Decade I | 5 | 2.27 | 1 | 0.01 | 1 | 0.12 | 1 | 0.26 | 1 | 0.60 | 1 | 1.28 | .... | ...... |
| Decade II | 3 | 3.06 | 1 | 0.03 | .... | ...... | 1 | 0.33 | .... | ...... | .... | ...... | 1 | 2.70 |
| Decade III | 3 | 1.46 | 1 | 0.08 | .... | ...... | .... | ...... | 2 | 1.38 | .... | ...... | .... | ...... |
| *August.* | | | | | | | | | | | | | | |
| Decade I | 3 | 0.79 | 1 | 0.04 | .... | ...... | 2 | 0.75 | .... | ...... | .... | ...... | .... | ...... |
| Decade II | 3 | 0.49 | 1 | 0.03 | 1 | 0.19 | 1 | 0.27 | .... | ...... | .... | ...... | .... | ...... |
| Decade III | 3 | 0.28 | 2 | 0.05 | 1 | 0.23 | .... | ...... | .... | ...... | .... | ...... | .... | ...... |

46. If this number, expressing according to international agreement the frequency of rainfall, were of any practical importance it ought to convey some definite idea as to the rainfall actually experienced.

47. The table of observed data shows that this number does not represent any such idea, for the actual rainfall during the five decades in question varied exceedingly. During the first decade an extraordinary rain of 2.70 inches fell, while during the last decade there were only two sprinkles, aggregating 0.05 inch, and one shower of 0.23 inch.

48. The total amount of rainfall in these five consecutive decades

gradually diminished from 3.06 inches in the first, through 1.46, 0.79, and 0.49, to 0.28 in the last decade.

49. Even taking both numbers combined, representing frequency and amount, we can obtain no definite idea from them as to the actual rainfalls that occurred. The really notable feature of the first decade is its one very heavy rain of 2.70 inches; the notation in universal use gives us no knowledge of this, the only important fact.

50. Accordingly, the notation or mode of record in use must be replaced by some other method that gives a fair and definite indication of the actual distribution of the rainfall that has occurred.

51. It is perfectly clear to every mathematical mind that the mere statement of number of rains (frequency) and total amount must be almost useless; for when merely the sum and the entire number of quantities are given, these individual quantities themselves remain absolutely indefinite and, therefore, unknown.

52. It may sound paradoxical, but it is not the less true, that when meteorologists in their monthly and yearly tables have entered their rainfall measurements under the two universally adopted headings of frequency and amount, they have practically almost wiped out the observations made. If the actual observations had been a thousandfold different from what they really were, the same final frequency and amount might have been obtained.

53. I was fully aware of this difficulty more than ten years ago. It was forced upon my mind, not from a purely scientific standpoint, but by my daily observation of the effects of rains in my garden and on my fields. The completion of the record of the observations of almost every new month added to my perplexity; the tabulated results of my own careful rainfall observations and my personal observations in garden and field did not harmonize. From light rains I observed good results, and by abundant rains often only great damage was done.

54. Most regretfully I admitted to myself that our scientific method of procedure in one of the most important subjects is not correct, and by using a faulty method of statement practically destroys the scientific work done.

55. In those days I spent considerable time in my large garden, over a mile away from town, and surrounded by timber and fields, located almost on the bluffs of the Iowa River, above Iowa City. During showers I watched the passing phenomena from the porch of my little garden house or from within the house through window and open door. From the most characteristic clatter of the drops of an approaching shower on the millions of oak leaves in the distance, till the outpour of, at times, strings of water on the shingle roof close over my head, I most intently tried to absorb mentally the mechanism

of nature in order to discover some rational method of expressing the great phenomenon.

56. Many a shower and rainstorm I have also observed as intently while at home. My meteorological observatory proper, being a single tower room on the third floor, with large windows in each of the four free walls, and covered with an almost flat tin roof, gave me an admirable place for these observations. Also here I was almost surrounded by the very phenomenon which I felt it necessary to more completely comprehend in order that our scientific measurements might be made of greater practical utility and their results brought into harmony with the experience of the gardener and the farmer.

57. Although I had no self-recording rain gauge, I connected one of my rain collectors with a large graduated cylinder standing in one of the windows of this observatory. I could thus watch each passing shower quantitatively, and make a record of the fall of rain as function of time.

58. In this way I gradually obtained a clearer scientific insight into the mechanism of rainfall, somewhat in the following order:

59. Rains and showers differ exceedingly in intensity. The linear depth of water is not a direct measure of the intensity of any rain.

60. Showers giving 0.20, 0.30, 0.40 inch of rain, in about equal lengths of time, are not in their intensity as 2 to 3 to 4. The intensity of two rains of 0.50 and 0.60 differs but little, but one rain of 0.10 and another of 0.20 differ greatly in intensity though the actual linear difference in amount is the same, namely, 0.10 inch.

61. Equal increase of height does not at all indicate equal increase of rain intensity. Hence the scale of rain intensity can not be a scale of equal linear units of rainfall.

62. This most important conclusion set me more adrift than ever, since it at first sight seemed to destroy the value of all rainfall measurements made. But gradually I became conscious of the following general principle:

63. Equal increase in rain intensity demands an increase in rainfall directly proportional to the amount fallen. Thus, to increase the intensity of a 2-inch rain as much as a 1-inch rain is increased by 0.10 inch it will require the fall of 0.20 inch in the same length of time.

64. This is the well known characteristic of the increase of the logarithmic function. The fact that I had in other directions of scientific research been led to a very close study of this important function, no doubt aided me greatly in detecting this relation.*

*Not deeming it advisable to introduce any mathematical language in the text, except it be absolutely necessary, I give the general formula, expressing the above reasoning, in this foot note.

65. This general result may also be expressed in the following manner:

To discover rainfall laws it is necessary to plot all correlated quantities as dependent on the logarithm of rainfall amount, and not on the linear amount of rainfall.

66. In all graphical investigations into rainfall laws it will therefore be necessary to take as abscissa the logarithm of the rainfall amount, and not the linear value of the rainfall itself.

In other words, if the intensity of rainfall increases as the arithmetical progression 1, 2, 3, 4, 5, 6, . . . ., the rainfall depth or amount must increase in a geometrical progression like 1, 2, 4, 8, 16, 32, . . . . .

67. If these figures stand for inches, then the 1-inch addition to the intensity 1 represents a unit of increase in the intensity of the rain 1. But to obtain the next increase of one unit in intensity requires 2 inches of rain. To secure the third unit of increase of intensity will require an addition of 4 inches of rain. The fourth unit of intensity can only be reached by an addition of 8 inches of rain.

68. It will now be readily understood that no general rainfall laws could be discovered so long as only equal linear depths of rainfall were taken into consideration as equal units of rain intensity.

69. The ratio of progression need not be two; but I adopt it as the simplest possible. This gives the following series of rain intensities and the corresponding amount in linear depth of water:

| | | | | | |
|---|---|---|---|---|---|
| Rain intensity | 1 | 2 | 3 | 4 | 5 |
| Rain amount | $2^0$ | $2^1$ | $2^2$ | $2^3$ | $2^4$ |
| Or | 1 | 2 | 4 | 8 | 16 |

70. It remains yet to select a convenient *unit* of rain intensity. After very careful consideration I have adopted the *tenth of an inch* as the unit of rain intensity. In English measure, the series of rain intensities is therefore

| | | | | | |
|---|---|---|---|---|---|
| Intensity | 1 | 2 | 3 | 4 | 5 |
| Amount (inches) | 0.1 | 0.2 | 0.4 | 0.8 | 1.6 |

71. For all practical purposes involved here it will suffice to take the metrical equivalent of the inch at 25 millimeters. Hence, the metrical intensity unit of rainfall is a quarter of a centimeter.

---

Let $h$ be the height of rainfall and $t$ the time, then the intensity of the rain at this time is

$$\frac{d\,h}{d\,t} = k\,h \qquad (1)$$

where $k$ is not dependent on $h$. Hence,

$$\log.\ h = k\,t \qquad (2)$$

where $k$ is constant in reference to $h$, but is a function of the other conditions of the phenomenon.

The result is that not $h$ but log. $h$ is the variable to be taken in rainfall investigations.

72. The following table represents the metrical rain intensity scale, practically identical with the English scale given above:

| | | | | | |
|---|---|---|---|---|---|
| Intensity | 1 | 2 | 3 | 4 | 5 |
| Amount (quarter-centimeters) | 1 | 2 | 4 | 8 | 16 |
| (Millimeters) | 2.5 | 5 | 10 | 20 | 40 |

73. Intensities less than the unit adopted (namely, one-tenth inch or one-quarter centimeter), are practically insignificant sprinkles and the lightest of showers.

74. The first three rain intensities represent almost entirely the rains useful to the gardener and farmer; they may, therefore, be called the *useful rains*, or the farmers' rains.

75. Rains of intensity 4 and 5 are very important in the economy of nature, and have powerfully contributed to the shaping of the present land surface and drainage system; they continue to be almost the only rains specially to be considered by hydraulic engineers and in the construction of roads and bridges.

76. But to the agriculturist these heavy rains are generally damaging, washing the rich soil from his fields and causing floods.

77. In this manner we have obtained the following

*Scale of rain intensities.*

| Intensity. | Degree. | Amount of rainfall. |
|---|---|---|
| 0 | Insignificant rains or sprinkles | Less than one-tenth inch (or one quarter-centimeter). |
| | USEFUL RAINS, FOR GRASS, GRAIN, AND CORN. | |
| 1 | Showers | One-tenth inch (or one quarter-centimeter) up to next degree. |
| 2 | Rains | Two-tenths of an inch (or two quarter-centimeters) and upwards to next degree. |
| 3 | Soaking rains | Four-tenths of an inch (or four quarter-centimeters) and upwards to next degree. |
| | EXCESSIVE, OR DAMAGING RAINS. | |
| 4 | Washing rains | Eight-tenths of an inch (or eight quarter-centimeters) and upwards to next degree. |
| 5 | Flooding rains | Sixteen-tenths of an inch (or sixteen quarter-centimeters) and upwards. |

78. Higher degrees occur so rarely that it is not necessary to include them in this general table of intensities.

Thus, during my twenty years of rainfall observations, I have recorded only one single rain exceeding 4 inches, and accordingly belonging properly to the sixth degree of intensity, though the total number of rains was 2,135 in that period of time, the unit of time being one day. This rain fell during the third decade of August, 1874, and amounted to 4.49 inches.

79. It should not be overlooked that, theoretically, any other unit of length might be adopted instead of the tenth-inch unit; also other modes of progression than by duplication could be used. But when all practical effects of rains and the relations of the two great systems of measure are taken into account, I believe that the unit and ratio adopted by me will be retained as satisfactory.

80. In the beginning of this research, when only the necessity of the logarithmic law had yet been recognized by me, I naturally adopted the *inch* as one of the units and two as the ratio. Thus I obtained two inches upwards and half and quarter inch downward.

81. Finally, as any general relation would of necessity be independent of all arbitrary units, I also retained the conventional English units of a tenth and a hundredth of an inch.

82. This gave me the following points in a preliminary scale of rain intensities:

| Ratio | ten | | ; | | two | | |
|---|---|---|---|---|---|---|---|
| Amount (inch)...... | 0.01, | 0.10 | ; | 0.25, | 0.50, | 1, | 2 |
| Degree ............... | 0 | 1 | ; | 2 | 3 | 4 | 5 |

83. This scale has been used by me in my Weather Reports since about 1886, excepting that the values counted for the quarter inch were not published. (See most of my Iowa Weather Reports, on pages 211 and 216 of each volume.)

84. When now taking up the more exhaustive study of my rainfall observations, I naturally retained the scale already used by me years ago. Hence, the tables compiled from my observations give the rains grouped according to this *full-inch scale.*

85. It is very easy and perfectly sufficient to reduce the resulting values from this preliminary full-inch scale to the general tenth-inch scale adopted.

86. The data taken from India, being in English units, have been reduced to the preliminary full-inch scale; but all data from countries using the metrical scale have been reduced to conform to our general tenth-inch scale, which is practically identical with the quarter-centimeter scale.

87. It may be objected that such an analysis of rainfall requires too great an effort and will occupy too much space. But this objection is without weight.

88. In the first place, it is a fact that we find time and place to record and tabulate regular observations of wind direction and force. This involves eight directions and their intensity, together with the number of calms. In tables, this involves seventeen columns of data.

89. The above analysis of rainfall, distinguishing six degrees of intensity, involves only *six* columns if we restrict ourselves to the analysis of frequencies, and only twelve columns when taking also the absolute amounts, with the totals of amount and number, the *complete* analysis of rainfall thus involves fourteen columns only.

90. Now if we can devote seventeen columns to the wind, will it not be possible to devote twelve or fourteen columns to the rain? Is not the rainfall of infinitely greater importance to the farmer—and to almost all people—than wind direction and force?

91. But really this comparison by columns does not present a fair

idea of the work involved. For the wind columns are all filled, and are threefold each, representing the three observations a day, while the rain columns are single, with but few places filled therein, because the number of rain days is only from one-fourth to one-third of the entire number of days.

92. Besides, it should be understood that the mere tabulation of number or frequency by intensity will give a very good expression of the observed amounts of rainfall also. Thus it will be quite possible to limit ordinary work to six columns, so far as rainfall is concerned.

93. In conclusion, it may be stated that the final result for any given period can always be expressed by two single numbers, the constants of the formula for the particular period in question. From the general formula, and by these two constants, all other values can then be calculated. The entire data of observation can also be reproduced by construction from the same constants.

94. Thus, these two final constants do not wipe out the data observed, but they represent them. The real data observed have not become the unknown roots of an indeterminate equation, but they are the formal elements of the totality of all the data observed, allowing the instant reproduction of the individual values either by calculation or by construction.

95. In concluding this chapter, it will hardly be necessary to state that I am fully aware that no detailed experimental demonstration of the principles stated has been here presented. I simply have endeavored to set forth the fundamental principle of my investigation of rainfall.

96. In the chapters that follow the laws will be deduced from the observed data by arranging them according to the logarithmic scale of intensities here established by reference to my long-continued study of the rain phenomenon.

97. The character and value of the laws so established will furnish a perfectly satisfactory empirical demonstration of the correctness of the intensity scale adopted, and prove conclusively that not the linear depth of rain but the logarithm of this depth is the physical measure of its intensity.

### TIME UNITS.

98. The duration of a shower is not a suitable time unit, lacking the condition of equality. In conformity with almost universal practice, the *day* of twenty-four hours is taken as time unit in this research. In subsequent studies of peculiar rain phenomena we shall use shorter time units also; but here it is of fundamental importance * to rigidly adhere to the day as time unit.

* The series of observations by Prof. Parvin, extending ten years back of my own observations, it has been found impossible to utilize in this research because this time unit had not been adhered to. See paragraph 318.

99. For the study of the yearly period I have found the *decade of ten days* the most useful. It has been made the greater time unit in all my previous meteorological work, as may be seen from my Iowa Weather Review, 1875, and the series of my Iowa Weather Reports, 1876–1888. At some other time I may set forth the great advantages of this ten-day unit; here it will not be necessary, and the results obtained will abundantly justify the choice made.

100. The next higher units are the *calendar year* and the *decennary* or ten-year period.

The introduction of the lustrum I believe practically to be a mistake for useful results, the period being too short.

The month will also be used in deference to common practice, though the scientific value of monthly means is very slight.

## II.—THE LAW OF TOTAL RAIN FREQUENCY.

101. By the total number of rain days we designate the number of days in any specified period of time on which rain fell to the amount of the rain intensity specified, or more. This number accordingly represents the total frequency of that intensity.

102. The data observed by me at Iowa City show that for the decennary periods from 1871 to 1880 and from 1881 to 1890, the total number of rain days for each given rain intensity, according to the preliminary full inch system (see paragraph 82), was as follows:

*Total number of rain days per year at Iowa City, Iowa.*

| Rain intensity.. | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Rain height *... | 1 | 10 | 25 | 50 | 100 | 200 |
| 1871 to 1880 ... | 101.6 | 60.4 | 39.8 | 23.2 | 10.9 | 2.4 |
| 1881 to 1890 ... | 111.9 | 68.0 | 40.6 | 22.3 | 9.1 | 1.9 |
| 1871 to 1890 ... | 106.8 | 64.2 | 40.2 | 22.8 | 10.0 | 2.1 |

103. The actual signification of these numbers will hardly need further explanation. Thus the total number of days on which the rain intensity was two or more averaged 39.8 during the decennary period beginning 1871, while during the next ten years it was almost the same number, 40.6 per annum; for the entire twenty years from 1871 to 1890 there were annually 40.2 days on which there fell 0.25 inch or more of rain.

104. According to this table there were 64.2 rain days per year of 0.10 inch rain or over; hence there were 64.2 less 40.2, or exactly twenty-four days a year on which the rainfall equaled or exceeded 0.10 inch without reaching 0.25 inch.

105. The twenty year's average of rain days bringing 2 inches or more of rain is 2.1 for Iowa City; for rain days bringing 1 inch or more of rain the total number is ten a year. Consequently, the

*In hundredths of an English inch, which we will call *centi-inches*.

number of rain days bringing 1 inch or more, but less than 2 inches of rain, was eight.

106. The data of observation in the little table given above (in paragraph 102), will now certainly be fully understood. They represent nothing but the results of all daily rain measurements made by me during the twenty years from 1871 to 1890 at Iowa City, and grouped according to the rain intensities defined by the daily rainfalls of 1, 10, 25, 50, 100, and 200 hundredths of an inch.

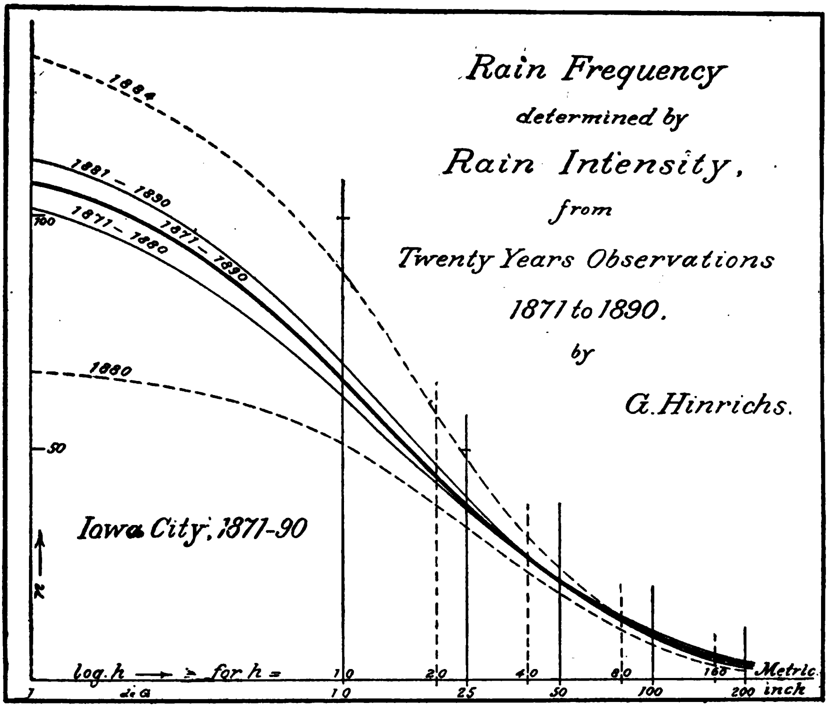


FIG. 1.

107. I may be permitted to call special attention to one very striking fact which this table of observed data forcibly brings into view. It is the almost absolute identity of the total number of rain days for the two decennary periods and for each one of the rain intensities. This is especially so for the higher intensities.

108. Thus, for the total number of rain days bringing 0.25 inch or more of rain the ten years' means differ only by one per cent from the twenty years' mean. I have no doubt that this apparent constancy of our rain frequency would not have been anticipated, considering the current opinion of the extraordinary variability of all rainfall data demanding long series of observations to reach such constancy of results.

109. To search for a general law in these figures of total rain fre-

quency they must be plotted as ordinates to the logarithm of the corresponding intensity as abscissæ, according to the general principle established in the first chapter.

110. Retaining the convenient unit of length—the one hundredth of an inch—used in the English system, we shall have from the tables of logarithms:

| Intensity. | Height, $h$. | Log. $h$. | (Log. $h$)$^2$. | (Log. $h$)$^3$. |
|---|---|---|---|---|
| 0 | 1 | 0.000 | 0.00 | 0.00 |
| 1 | 10 | 1.000 | 1.00 | 1.00 |
| 2 | 25 | 1.398 | 1.96 | 2.74 |
| 3 | 50 | 1.699 | 2.89 | 4.91 |
| 4 | 100 | 2.000 | 4.00 | 8.00 |
| 5 | 200 | 2.301 | 5.29 | 12.17 |

The square and cube of the logarithm have been added for use in subsequent paragraphs (115 and following).

111. Fig. 1 shows the resulting curves for the two decennial periods and for the entire twenty years. I have also drawn the curves for the single years of 1880 and 1884, which mark the range of all the years recorded.

112. These curves are perfectly definite, evidently the geometrical expression of some definite mathematical function embodying the unknown law of total rain frequency.

113. I immediately recognized in these curves old friends that I had met in different branches of science. More than twenty years ago I subjected this class of curves to a very extended, strictly empirical study, and was gratified by the discovery of a most simple and direct mathematical expression of the same, which will be given further on. (See Chapter IV.)

114. Applying "The Method of Quantitative Induction in Physical Science," published by me twenty years ago,* I discovered that the logarithm of the number of rain days forms a parabola of the third power with the logarithm of the corresponding rain depth $h$.

115. That is, if we take as abscissa the cube of the logarithm of the height $h$, and as ordinates the logarithms of the corresponding total number of rainy days, we shall obtain a straight line. This is shown in Fig. 2, which has been constructed by making use of the cube of the logarithm of $h$, tabulated in paragraph 110.

116. Accordingly we have, if $n$ be the total number of rain days on which the rainfall equaled or exceeded $h$, hundredths of an inch (English),

(3) $y = \log. n.$

(4) $x = (\log. h)^3.$

(5) $y = a - b\,x.$

* Davenport and Leipzig, 1872.

That is, $x$ and $y$ are the co-ordinates of points in one and the same straight line, the position of which is perfectly determined by the constants $a$ and $b$.

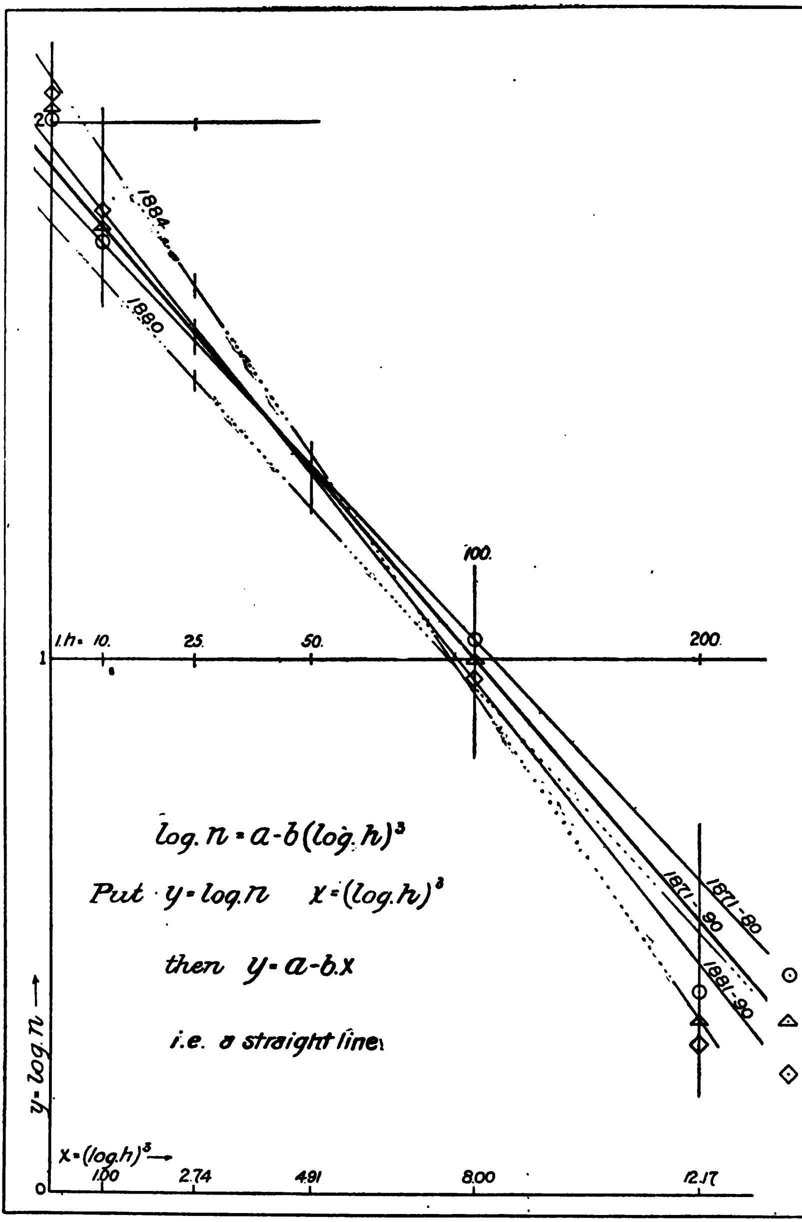


FIG. 2.—Graphic determination of constants for Fig. 1.

117. The observed values of $n$ give by common three-place tables of logarithms the corresponding values of $y$. The values of $x$, the

cube of log. $h$, are tabulated in paragraph 110. Consequently, by adopting any convenient and sufficiently large scale, those values of $x$ and $y$ can be plotted, giving a series of points.

118. Now these points should all lie in one straight line, which then readily would give us the values of the constants in formula (5), after which we could calculate the number, $n$, of rain days for any rainfall, $h$, by the equivalent equation

$$(6) \qquad \log. n = a - b\,(\log. h)^3.$$

119. In drawing the straight line passing closest to all the points given it must be borne in mind that the points corresponding to $h = 10$, 25, and 50—our farmer's rains—are the most important also mathematically, and only the smallest deviations from these points should be admitted, though thereby the straight line will have to leave the other points farther to the right or to the left.

120. It is evident that the point corresponding to $h = 1$ is neither of great intrinsic value nor can it be said to be accurately known.

121. It is of hardly any intrinsic value, because the rains represented are themselves insignificant; in fact, $h = 0.01$ inch being the very limit of the recognized rainfall in the English-speaking countries.

122. But for that very reason the number of rain days corresponding to this amount of water is not at all accurately known. The tendency of most observers will be to "call it a hundredth of an inch" and record it as such whenever the rain shower comes any way near this limit of measure.

123. In other words, almost all observers, reading the rainfall to the hundredth of an inch, will considerably overstate the number of rains of this limiting magnitude.

124. I notice I have done this very thing. I find by calculation that during my twenty years' observations there have been eighty-four rains equal and exceeding this hundredth of an inch limit a year. My tables show that I recorded one hundred and seven such rains per year.

125. I have no doubt that there were actually only the calculated eighty-four such rains a year, and that twenty-three light sprinkles each year only approached this limit of rainfall without quite reaching the same.

126. The points corresponding to $h = 100$ and 200 are also of less importance in drawing the straight line, because apparently large distances of the straight line from these points represent in fact only a small deviation in the corresponding number, $n$, observed and calculated. This is due to the *cube* of log. $h$ being taken as the abscissa.

127. I must be allowed to insist on this graphical method of determining the values of the constants in my formula. The professional mathematician may propose to use elaborate calculations, based upon

the method of the least squares, for this determination. Experience has shown me that in all cases of this kind the graphical method is not only infinitely less laborious, but at the same time yields more correct results, and especially avoids egregious blunders. (Compare paragraphs 259 to 266.)

128. In the original drawing, photo-engraved in Fig. 2, the scale used for $y = \log. n$ was 10 inches as unit, so that the second place in the logarithm was represented by the tenths of an inch, and the third place by the hundredths of an inch. The drawing consequently faithfully represented the logarithm of $n$ to three places, which is quite sufficient.

129. The unit of $x$ or the cube of log. $h$ was one inch, so that the values tabulated in paragraph 110 were also exactly represented (to two places).

130. Drawing a straight line near to the points corresponding to $h = 10$, 25, 50, and next to these to $h = 100$, cuts off the value of the constant $a$ on the zero line on the left. Having also drawn the ordinate at $x = 10$ inches, we get by difference the value of ten times the constant $b$, and accordingly have determined $b$.

131. Thus plotting the points for the twenty years' average for 1871 to 1890, we find that the points corresponding to $h = 10$, 25, 50, and 100 are* as nearly in a straight line as a ruler with transparent edge allows us to see. Drawing this line, we find it cuts off the ordinate 19.25 inches at $x = 0$ and 7.67 at $x = 10$ inches. That is, $a = 1.925$ and in the distance 10 units of $x$ the logarithm of $n$ sinks from 1.925 to 0.767, or the amount of 1.158, which is ten times the value of $b$. Hence $b = 0.1158$, for which we may take 0.116.

132. By means of these values of the constants $a = 1.925$ and $b = 0.116$ the simple formula (6) of paragraph 118 will, by using the values of (log. $h$) tabulated in paragraph 110, give the following caculated values of $n$:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
| $n$ calculated | 84.1 | 64.4 | 40.6 | 22.8 | 10.0 | 3.3 |
| $n$ observed | 106.8 | 64.2 | 40.2 | 22.8 | 10.0 | 2.1 |
| Cor. of calculated value. | + 22.7 | — 0.2 | — 0.4 | 0.0 | 0.0 | —1.2 |

133. Remembering that the record naturally greatly overstates the limiting number of rains for $h = 0.01$ inch, *we may assert that the calculated values are identical with the observed values.* Most assuredly the trifling differences are entirely inside of the precision of the empirical determination of the observed numbers.

134. Consequently, the formula (6), paragraph 118, expresses the law of nature according to which the number of days of rainfall depends upon the amount of rain falling.

*For 25 and 50 they are not specially marked by circle, triangle, or square in Fig. 2, being too close together.

135. The two constants $a$ and $b$, of our formula, thus completely represent the actual distribution of the rainfall. They allow us to determine the total number of rain days for *any* given amount of rainfall. They accordingly are precisely such two values as we require for the comprehensive representation of the actual distribution of the rainfall. (See paragraphs 93 and 94.)

136. These two constants will, necessarily, in time take the place of amount and frequency now used in meteorology. (See paragraph 52.)

137. Let us, for example, determine the total number of rain days for the same series of observations but in relation to the metrical units which I have proposed for universal adoption.

138. The constantly required values for the metrical scale are presented in the following table:

| Intensity. | $h$. | | Log. $h$. | (Log. $h)^2$. | (Log. $h)^3$. |
|---|---|---|---|---|---|
| | *mm.* | *Centi-inch.* | | | |
| 0 | 0.3 | 1.2 | 0.08 | 0.01 | 0.001 |
| 1 | 2.5 | 10 | 1.000 | 1.00 | 1.00 |
| 2 | 5 | 20 | 1.301 | 1.69 | 2.19 |
| 3 | 10 | 40 | 1.602 | 2.56 | 4.09 |
| 4 | 20 | 80 | 1.903 | 3.61 | 6.86 |
| 5 | 40 | 160 | 2.204 | 4.84 | 10.65 |

Here $h$ is given in hundredths of an inch (centi-inch) as before. (Compare paragraph 110.)

139. The result of this calculation is: Total number of rain days at Iowa City, twenty-year mean, metrical scale:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ (centi-inch.)............ | = | 10 | 20 | 40 | 80 | 160 |
| Or (millimeters)... | | 2.5 | 5 | 10 | 20 | 40 |
| $n$ ........................... | = | 64.4 | 46.9 | 28.3 | 13.5 | 4.9 |

For $h = 1.2$ the calculation gives $n = 84.1$ insignificant rains.

140. Upon measuring the dotted ordinates corresponding to the metrical scale, it will be found that they agree exactly with the above calculated values.

141. By going over the entire body of observations and actually counting the number of rain days according to these metrical limits, I have no doubt, whatever, the numbers here found by calculation from our formula will agree closely with the new count. (See also paragraphs 132 and 133.)

142. The far reaching importance of such a general law connecting the rain frequency with rain intensity demands that it should be abundantly confirmed by data of observation. This we now proceed to do.

143. Taking the ten-year series from 1881 to 1890 at Iowa City, I find, in the manner fully explained, the values of the constants $a = 1.960$ and $b = 0.126$. These values give the following calculated

values of $n$, which are again practically identical with the values of $n$ observed tabulated in paragraph 102:

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $n$ calculated | 91.2 | 68.3 | 41.2 | 22.0 | 9.0 | 2.7 |
| $n$ observed | 111.9 | 68.0 | 40.6 | 22.3 | 9.1 | 1.9 |
| Correction of calc. | +20.7 | −0.3 | −0.6 | +0.3 | +0.1 | −0.8 |

It will be remembered that the insignificant rains for $h = 1$ are recorded too high.

144. For the decennary 1871–80, I find the constants $a = 1.890$ and $b = 0.1066$ with the following most satisfactory results:

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $n$ calculated | 77.6 | 60.7 | 39.7 | 23.3 | 10.9 | 3.9 |
| $n$ observed | 101.6 | 60.4 | 39.8 | 23.2 | 10.9 | 2.4 |
| Correction of calc. | +24.0 | −0.3 | +0.1 | −0.1 | 0.0 | −1.5 |

145. Thus far I have only considered ten or twenty years' means and have found the calculated values practically identical with those observed, always excepting the insignificant rains (for $h = 1$) which are necessarily recorded too high. (See paragraphs 123 to 125.)

146. But if it is truly a general law it ought to give reasonably close approximations to even the observations of a single year.

147. It will be particularly interesting to test the law in reference to exceptionally dry or wet years, as well as to the years 1880 and 1884, which have the lowest and highest general frequency ($h = 10$) observed.

148. For the year 1880, of least general frequency, I find the value of the constants to be $a$ 1.820 and $b$ 0.112, with the following results:

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $n$ calculated | 66.1 | 51.1 | 32.7 | 18.6 | 8.4 | 2.9 |
| $n$ observed | 97 | 51 | 33 | 22 | 8 | 1 |

The only notable disturbance which marks this year is the actual excess of about three rain days of half an inch rainfall each.

149. The year 1884, with the highest general rain frequency, gave the constants $a$ 2.095 and $b$ 0.146.

The calculated values agree very well with the observed, except for the quarter-inch rains, which were exceptionally deficient.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $n$ calculated | 124.5 | 88.9 | 62.4 | 23.9 | 8.5 | 2.1 |
| $n$ observed | 134 | 88 | 48 | 24 | 10 | 1 |

150. The year of drought, 1886, gives the constants $a$ 1.975 and $b$ 0.166, with the following comparisons of calculated and observed values:

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $n$ calculated | 94.4 | 64.4 | 33.0 | 14.4 | 4.4 | 0.9 |
| $n$ observed | 101 | 55 | 26 | 17 | 4 | 1 |
| Correction calc | | −9.4 | −7 | +2.6 | −0.4 | +0.1 |

This shows a notable relative deficiency in the lighter useful rains, 10 and 25.

151. The flood year, 1881, gives us *a* 2.000 and *b* 0.114.

| | | | | | | |
|---|---|---|---|---|---|---|
| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
| *n* calculated | 100 | 76.9 | 48.8 | 27.5 | 12.3 | 4.1 |
| *n* observed | 122 | 73 | 48 | 31 | 10 | 5 |
| Correction calculated | | −4 | −0.8 | +3.5 | −2.3 | +0.9 |

The most notable disturbance of a regular distribution was the deficiency of showers (10), and excess of soaking rains (50).

152. We thus find the law applicable even to the distribution of the rains of a single year at a single station, though here the notable individual features of the year present themselves not only in the special values of the two constants, but also in more or less marked disturbance of the regular distribution, which disturbance it becomes easy to express and record, as has just been shown.

153. The law thus valid must also be applicable to the stations of a given limited territory for a given period of time. It will answer here to show that it is fully pronounced in the rains of even a single month falling at a dozen stations in a single State.

154. Let us take the rain reports of the twelve pentad stations of Iowa for the month of May, 1881, from my Iowa Weather Report for that year. The constants are found to be *a* 2.030 and *b* 0.141.

| | | | | | | |
|---|---|---|---|---|---|---|
| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
| *n* calculated | 107.4 | 77.5 | 44.1 | 21.8 | 8.0 | 2.1 |
| *n* observed | 106 | 77 | 43 | 25 | 8 | 1 |

The only disturbance is the slight excess (three) of the half-inch rains. The observers during this month did not exaggerate the insignificant rains—most rains being very considerable.

155. The law (equation 6, paragraph 118) is accordingly well established by my observations. We might now begin the study of the variation of the constants with the character of the year. It is more important to complete the establishment of this law by proving that it is by no means local, peculiar to Iowa or the Mississippi Valley, but obtains equally in all countries and climes, so that in fact it is a universal law of rainfall.

## III.—THE LAW OF TOTAL FREQUENCY IN GENERAL.

156. It will manifestly be unnecessary to study reports from all countries in order to ascertain whether the law just established as a fact for Iowa is general. If we select regions of the globe differing greatly in their rainfall conditions and find the law valid in all, we can safely consider it proved to be universal.

157. Suppose we mentally make a journey from the coasts of India (Bombay, Madras, Calcutta) through its interior (Nagpur, Allahabad, Lucknow), passing by way of Lahore in the Punjab to dry As-

trakhan on the Caspian Sea. Let us from here go through Siberia (Irkutsk, Barnaul, Tobolsk) and European Russia (Kasan, Moscow, and Warsaw) to central Europe, visiting both its continental capitals (Berlin, Vienna) and the marine region Holstein (Hamburg, Kiel), and the neighboring stations on the North Sea (Borkum, Keitum) and the Baltic (Wustrow, Swinemunde).

158. If on all this tour we find our law conform to observation equally as well as in Iowa, we dare say that it is truly universal. Finally, we will also make a visit to Norway, the rainfall conditions of which are so exceedingly peculiar; but also even here we shall find our law manifest in every rain.

159. It will not be necessary for us to stay long in any one place. The record of a single year will be found quite sufficient, especially if we combine two to six stations to represent any one great rainfall region.

160. This remarkable circumstance adds greatly to the value of this law, proving it to be no mere general abstraction that becomes notable only in the course of many years, but a direct representation of the actual mechanism of all rainfall phenomena.

161. Thanks to the courtesy of the directors of the Indian, Russian, German, Austrian, Norwegian, and British weather services, I have the material for this investigation at hand in the reports of these services, which they have favored me with for years.

162. These reports, in accordance with the wise direction of the first international congress, contain the actual observations of a limited number of stations, and not merely the monthly summaries.* These latter would have been without any value whatever in this work.

163. The data taken from these original reports have been united in monthly and yearly summaries. They all represent the year 1889, with the exception of Berlin and Aachen, which are of 1888.

164. Here it is sufficient to give the yearly total number of rain days of each rain intensity for each station, together with the *mean* values of the two to six stations that are combined to represent each particular region.

165. The units used are the familiar English and metric units fully explained, having the $h$=10 centi-inches=one-tenth inch=2.5 millimeters in common.

*The publication of my own observations in the Iowa Weather Reports was strictly conformable to this direction. It is the only station in the United States for which such a publication has been made, I believe.

*Total number of rain days in* 1889, *arranged according to rain intensities.*

| 166. INDIAN EMPIRE. | English units (in centi-inches). | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 10 | 25 | 50 | 100 | 200 |
| Coast (three stations): | | | | | | |
| Calcutta | 128 | 87 | 63 | 41 | 13 | 5 |
| Madras | 84 | 52 | 33 | 23 | 12 | 5 |
| Bombay | 103 | 76 | 50 | 35 | 22 | 12 |
| Mean | 105.0 | 71.7 | 48.7 | 33.0 | 15.7 | 7.3 |
| Interior (three stations): | | | | | | |
| Nagpur | 87 | 56 | 42 | 25 | 15 | 2 |
| Allahabad | 85 | 54 | 39 | 21 | 9 | 3 |
| Lucknow | 70 | 55 | 44 | 32 | 19 | 4 |
| Mean | 80.7 | 55.0 | 41.7 | 26.0 | 14.3 | 3.0 |
| Mean for coast and interior (six stations) | 92.8 | 63.5 | 45.2 | 29.5 | 15.0 | 5.2 |
| Lahore | 40 | 28 | 23 | 17 | 6 | 3 |

| 167. RUSSIAN EMPIRE. | Metric units (in centi-inches). | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 10 | 20 | 40 | 80 | 160 |
| Astrakhan | 58 | 17 | 7 | 1 | 1 | 0 |
| Siberia (three stations): | | | | | | |
| Irkutsk | 100 | 41 | 13 | 4 | 1 | 0 |
| Barnaul | 127 | 64 | 32 | 14 | 2 | 0 |
| Tobolsk | 111 | 30 | 14 | 3 | 1 | 0 |
| Mean | 112.7 | 45.0 | 19.7 | 7.0 | 1.3 | 0.0 |
| European Russia (three stations): | | | | | | |
| Kasan | 120 | 47 | 21 | 10 | 1 | 0 |
| Moscow | 144 | 64 | 31 | 15 | 4 | 0 |
| Warsaw | 148 | 64 | 34 | 18 | 7 | 1 |
| Mean | 137.3 | 58.3 | 28.7 | 14.3 | 4.0 | 0.3 |
| Mean for Siberia and European Russia | 125.0 | 51.6 | 24.2 | 10.7 | 2.6 | 0.2 |
| 168. GERMAN EMPIRE. | | | | | | |
| Central Europe (two stations): | | | | | | |
| Berlin (1888) | 145 | 69 | 43 | 13 | 3 | 0 |
| Vienna | 146 | 76 | 44 | 16 | 7 | 0 |
| Mean | 145.5 | 72.5 | 43.5 | 14.5 | 5.0 | 0.0 |
| Aachen (1888) | 180 | 104 | 62 | 29 | 7 | 0 |
| Holstein— | | | | | | |
| Land (two stations): | | | | | | |
| Hamburg | 183 | 88 | 51 | 20 | 2 | 1 |
| Kiel | 183 | 78 | 48 | 18 | 2 | 0 |
| Mean | 183.0 | 83.0 | 49.5 | 19.0 | 2.0 | 0.5 |
| North Sea (two stations): | | | | | | |
| Keitum | 149 | 81 | 43 | 20 | 3 | 0 |
| Borkum | 136 | 89 | 51 | 23 | 9 | 0 |
| Mean | 142.5 | 85.0 | 47.0 | 21.5 | 6.0 | 0.0 |
| Mean for Holstein (four stations) | 162.7 | 84.0 | 48.2 | 20.2 | 4.0 | 0.2 |
| Baltic (two stations): | | | | | | |
| Wustrow | 126 | 63 | 38 | 17 | 2 | 1 |
| Swinemunde | 131 | 55 | 30 | 9 | 1 | 1 |
| Mean | 128.5 | 59.0 | 34.0 | 13.0 | 1.5 | 1.0 |
| 169. NORWAY. | | | | | | |
| East coast (two stations): | | | | | | |
| Christiania | 97 | 44 | 27 | 9 | 6 | 1 |
| Færder | 87 | 51 | 30 | 15 | 4 | 2 |
| Mean | 92.0 | 47.5 | 28.5 | 12.0 | 5.0 | 1.5 |

*Total number of rain days in* 1889—Continued.

| 169. NORWAY—*Continued.* | *h* Metric units (in centi-inches). | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 10 | 20 | 40 | 80 | 160 |
| **South coast (two stations):** | | | | | | |
| Mandal | 91 | 78 | 67 | 50 | 15 | 0 |
| Skudenes | 155 | 120 | 82 | 36 | 9 | 1 |
| Mean | 123.0 | 99.0 | 74.5 | 43.0 | 12.0 | 0.5 |
| **West coast or Bergen coast (two stations):** | | | | | | |
| Bergen | 157 | 118 | 104 | 64 | 27 | 6 |
| Floroe | 178 | 142 | 106 | 59 | 22 | 4 |
| Mean | 167.5 | 130.0 | 105.0 | 61.5 | 24.5 | 5.0 |
| Mean for South Atlantic coast (four stations preceding). | 145.2 | 114.5 | 89.7 | 52.2 | 18.2 | 2.8 |
| **North Atlantic coast (three stations):** | | | | | | |
| Christiansund | 171 | 92 | 57 | 30 | 5 | 3 |
| Bronoe | 145 | 92 | 58 | 24 | 7 | 1 |
| Bodoe | 156 | 85 | 58 | 27 | 5 | 0 |
| Mean | 157.3 | 89.7 | 57.7 | 27.0 | 5.7 | 1.3 |
| **Interior (two stations):** | | | | | | |
| Dovre | 113 | 35 | 21 | 4 | 0 | 0 |
| Alten | 79 | 34 | 19 | 10 | 3 | 0 |
| Mean | 96.0 | 34.5 | 20.0 | 7.0 | 1.5 | 0.0 |
| 170. IOWA CITY. (See paragraph 139). | | | | | | |
| Twenty years—1871 to 1890 | 84.1 | 64.4 | 46.9 | 28.3 | 13.5 | 4.9 |

171. First of all I have plotted these values in the manner fully explained, taking the observed number, $n$, as ordinate to the *logarithm* of the corresponding rainfall, $h$, as abscissa. Fig. 3 represents the resulting curves for the mean values of the regions specified. (Compare paragraphs 109 and 110.)

172. A mere glance at these curves is sufficient to show that they are of the same general form as those of Fig. 1, and that consequently our law applies to all of these climatologically most diverse regions of the globe.

173. The varying individual dimensions of this curve give a most impressive graphical representation of the climatic differences of these regions.

174. The two most extreme cases represent the Bergen coast (Bergen and Floroe) and Astrakhan on the Caspian Sea. The curve marked Holstein represents four stations (Hamburg, Kiel, in Holstein proper, and Keitum and Borkum are near islands in the North Sea). The curve marked Russia represents the mean of six stations, namely, three in Siberia (Irkutsk, Barnaul, and Tobolsk) and three in European Russia (Kasan, Moscow, and Warsaw).

175. It will be noticed that the curve representing the rainfall of tropical India cuts all the others; while the number of rains of low intensity is low, the number of rains of high intensity is seen to be greatest in this tropical country.

176. The line marked Russia is most typical of continental climate,

The number of insignificant rains is large, and the total number of higher rains rapidly diminishes with increasing intensity, being practically zero for rains of 40 millimeters or 160 centi-inches.

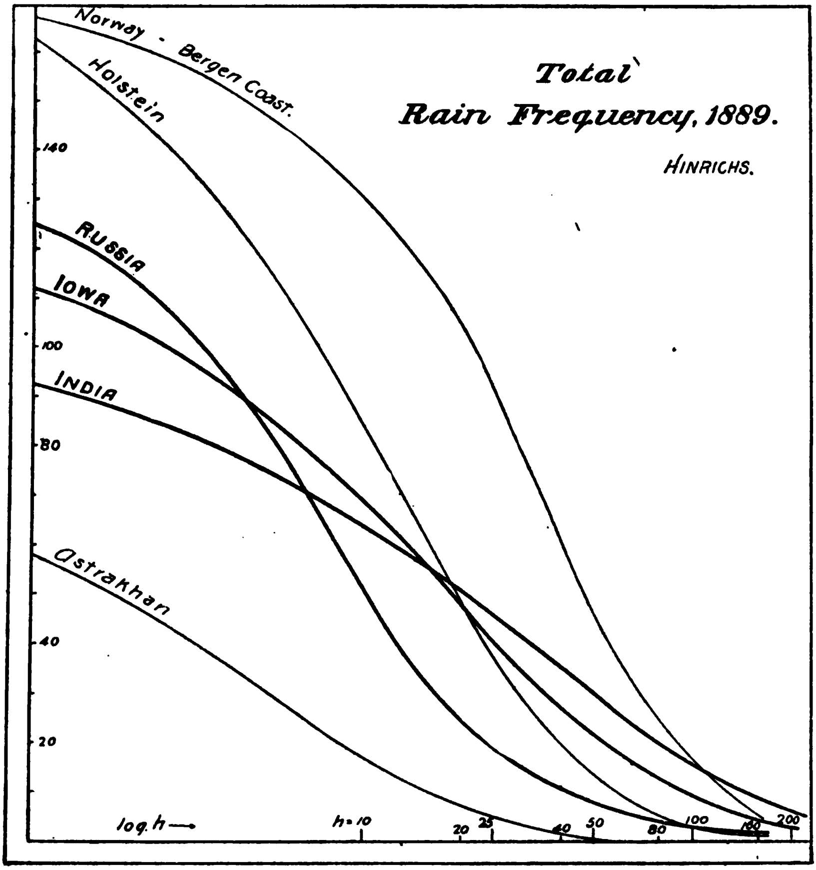


FIG. 3.

177. The curve Holstein, representing northwestern Germany and the adjacent North Sea, is of exactly the same general character as the curve marked Russia, but it cuts every ordinate at a higher point until the rainfall of 40 millimeters is reached.

178. On the contrary, the curve Iowa does not possess the general character of the continental line marked Russia. In fact, it has a form of its own, uniting the characteristic features of both Siberia and India. For rains of high intensity Iowa is nearest India, while for rains of low intensity it resembles Siberia.

179. The mountain barrier between India and Siberia is well marked in the notable contrast of form of the curves named India and Russia. In America we have no such barrier; the Mississippi Valley is open to the Gulf of Mexico and also to the Arctic plains.

Hence, our Iowa rain curve unites the features of the curves of India and Siberia. We have heavy rains like those of India, but not as many. We have sprinkles like Siberia, but not as many. We have showers and rains, useful to the farmer, but many more than Russia; though not as many as India, we have abundantly sufficient in normal seasons.

The following table, giving the calculated rain frequency for the insignificant and the first and last useful rains, brings out both resemblances and contrasts very strikingly:

| | | | |
|---|---|---|---|
| $h$ | 1 | 10 | 40 |
| India | 78.0 | 64.0 | 29.0 |
| Iowa | ...... | 64.4 | 28.3 |
| | 84.1 | ...... | ...... |
| Russia | 87.5 | 52.4 | 10.7 |

180. In order to test my law quantitatively we must determine the constants $a$ and $b$ and compare the values of the number, $n$, of the rain days calculated from our formula with those observed. (See formula 6, paragraph 118.)

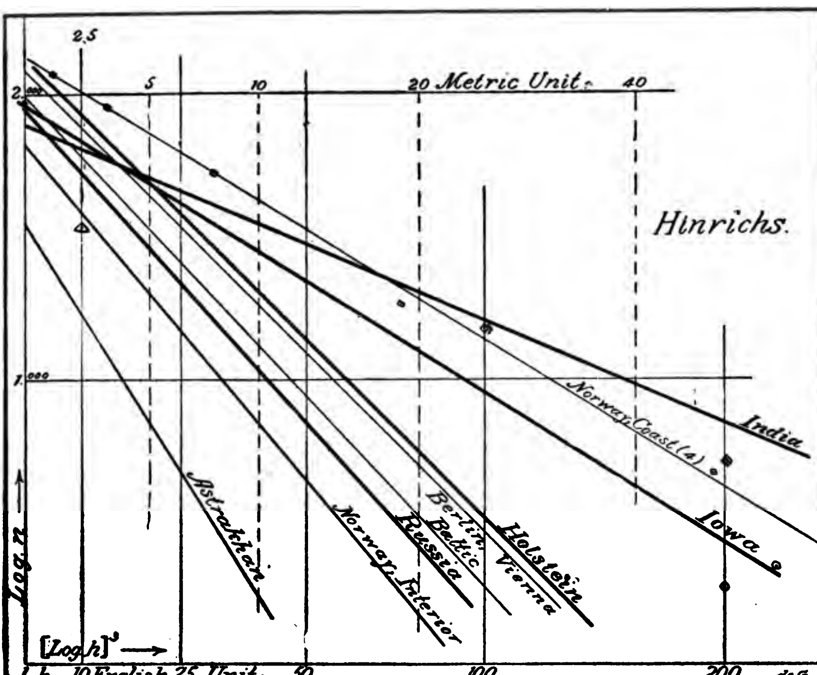


Fig. 4.—Graphic determination of constants for Fig. 3.

181. In order to determine the constants I have plotted the values of the logarithm of the observed number of rain days, $n$, as ordinates to the cube of the logarithm of the rainfall, $h$, as abscissa. (Compare paragraph 117.)

Then I draw a straight line that passes the nearest through these points, giving greatest weight to the points corresponding to the agricultural rains, $h = 10, 25, 50$, as fully explained in preceding paragraphs (119 *et seq.*).

182. Of the lines so drawn Fig. 4 gives the most important ones. In the original drawing the unit of log. $n$ was 10 inches, so that the third place of the logarithm will be quite well represented in hundredths of an inch. The unit for the cube of log. $h$ was 2 inches, so that the second decimal of the abscissa is accurately represented in fiftieths of an inch. Such a drawing,* carefully made, gives immediately the most probable values of the constants sought.

183. Before we shall take these constants from Fig. 4 we would invite special attention to the climatic features represented most strikingly in this diagram. In fact, for climatological study this figure is much more serviceable than Fig. 3.

184. The straight lines representing the distribution of rainfall for Holstein (northwest Germany and North Sea), central Germany (Berlin and Vienna), the Baltic, and Russia (European and Asiatic) are nearly parallel and quite strongly inclined. From northwestern Germany to Siberia the distribution of rain days according to rain intensity is therefore practically the same, only the *total* number of rains of each intensity diminishing gradually from the North Sea eastward to Irkutsk.

185. The straight line representing India is much less inclined, so that the heavier rains diminish in number much less rapidly in this tropical country than on the continent proper.

186. The position of Iowa, intermediate between India and Siberia, is more strikingly shown by this diagram than even by Fig. 3. The line Iowa almost bisects the angle formed by the lines of India and Holstein, which latter is nearly parallel to that of Russia.

187. This shows most unmistakably that the rainfall of Iowa, and of the entire Mississippi Valley, is tropical, like that of India, so far as the heavy summer rains are concerned, and also continental like that of Siberia for the light rains, being less extreme than that of either of these contrasting countries.

188. We will now take the constants $a$ and $b$ from each of the lines in Fig. 4 and compare the observed values, $n$, with those calculated by means of our formula (6), paragraph 118, using the constants so determined. We will also add the correction which would have to be applied to the calculated values to produce the observed values.

* The original drawing is in *fine lines*, using different colored inks for the different points and lines of each region. The drawing photo-engraved was a copy of the original in *heavy* lines and black, to secure a good photo-engraving.

189. The line representing the six stations of India gives $a$ 1.892 and $b$ 0.086, with the following results:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
| $n$ calculated | 78.0 | 64.0 | 45.4 | 29.0 | 16.0 | 7.0 |
| $n$ observed | 92.8 | 63.3 | 45.2 | 29.5 | 15.1 | 5.2 |
| Correction calculated | +14.8 | —0.7 | —0.2 | +0.5 | —0.9 | —1.8 |

Omitting the insignificant rains, for reasons often stated, we find the agreement very satisfactory.

190. The straight line, representing equally three stations of Siberia and as many in European Russia, gives $a$ 1.942 and $b$ 0.223 for the Russian Empire. We find, for

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 20 | 40 | 80 | 160 |
| $n$ calculated | 87.5 | 52.4 | 28.4 | 10.7 | 2.6 | 0.4 |
| $n$ observed | 125.0 | 51.6 | 24.2 | 10.7 | 2.6 | 0.2 |
| Correction calculated | +37.5 | —0.8 | —4.2 | 0.0 | 0.0 | —0.2 |

Here is only one notable deviation: for $h$ 20 centi-inches the number of rain days was too low, a seasonal peculiarity for the year 1889.

191. The two capitals of central Europe, Berlin (1888) and Vienna (1889), give $a$ 2.078 and $b$ 0.2006, with the following values of $n$:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 20 | 40 | 80 | 160 |
| $n$ calculated | 119.7 | 75.3 | 43.6 | 18.1 | 5.0 | 0.9 |
| $n$ observed | 145.5 | 72.5 | 43.5 | 14.5 | 5.0 | 0.0 |
| Correction calculated | +25.8 | —2.8 | —0.1 | —3.6 | 0.0 | —0.9 |

The values for 10, and especially for 40 centi-inches are too low; the latter cannot possibly be brought into line, so that rains of 40 centi-inches were particularly scarce. As the years considered are not the same, the case is of less importance.

192. For the four stations grouped under Holstein, we find $a$ 2.129 and $b$ 0.202.

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 20 | 40 | 80 | 160 |
| $n$ calculated | 134.6 | 84.5 | 48.6 | 20.1 | 5.5 | 0.95 |
| $n$ observed | 162.7 | 84.0 | 48.2 | 20.2 | 4.0 | 0.2 |
| Correction calc | +28.1 | —0.5 | —0.4 | +0.1 | —1.5 | —0.7 |

When we consider that we here have but four stations for one single year, and also remember the great difference in the situation of these stations, the agreement is most remarkable indeed.

193. The station Lahore, in the Punjab, gives $a$ 1.500 and $b$ 0.055, with the following results:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
| $n$ calculated | 31.6 | 27.9 | 22.3 | 17.0 | 11.5 | 6.8 |
| $n$ observed | 40 | 28 | 23 | 17 | 6 | 3 |
| Correction calculated | +8.4 | +0.1 | +0.7 | 0.0 | —5.5 | —3.8 |

For a single year and so peculiar rain conditions the agreement is most striking, except for the last two intensities, which we shall consider again further on.

194. Astrakhan is also climatologically very remarkable. We find $a$ 1.55 and $b$ 0.32, with the following results:

| | | | | | |
|---|---|---|---|---|---|
| $h$ | 1 | 10 | 20 | 40 | 80 |
| $n$ calculated | 35.5 | 17.0 | 7.1 | 1.7 | 0.2 |
| $n$ observed | 58 | 17 | 7 | 1 | 1 |
| Correction of calculation | +22 | 0.0 | —0.1 | —0.7 | +0.8 |

Excepting always the insignificant rains, we find our formula represents dry Astrakhan as perfectly as it did tropic Bombay.

195. The two stations in the interior of Norway, namely Dovre and Alten, give $a$ 1.82, $b$ 0.24, and

| | | | | | |
|---|---|---|---|---|---|
| $h$ | 1 | 10 | 20 | 40 | 80 |
| $n$ calculated | 66.1 | 38.0 | 19.7 | 6.9 | 1.5 |
| $n$ observed | 96.0 | 34.5 | 20.0 | 7.0 | 1.5 |
| Correction calculated | +30 | —3.5 | +0.3 | +0.1 | 0.0 |

The correction is only notable for the light rains of 0.10 inch, which were deficient this year (1889).

196. The position on Fig. 4 of the line for Norway's interior is almost identical with that of Siberia, showing true continental rain conditions in the interior of the comparatively narrow peninsula.

197. For the coast stations of Norway, the points determined by the values of $y = \log. n$, and $x = (\log. h)^3$ do not lie in a straight line. In other words, there must be special conditions modifying the general law.

The mountain range close to the Atlantic causes more abundant precipitation than would occur without this barrier. We can take this special physical condition into calculation by employing the general principle of the variation of constants.

198. Instead of being really constant, the quantity $b$ will vary with the rain intensity, log. $h$. The simplest manner of this variation will be proportional to the intensity, or

$$(7) \qquad b = c^1 \log. h$$

where $b^1$ is a new constant.

199. The formula (6) thereby changes to

$$(8) \qquad \log. n = a - c^1 (\log. h)^4$$

which quite satisfactorily represents the distribution of the rains according to their intensity along the coast regions of Norway, and will, no doubt, express the corresponding special conditions in Chile, Portugal, and elsewhere.

200. For the north coast, comprising the stations Christiansund, Bronœ, and Bodœ, we find $a$ 2.022 and $b$ 0.0904, giving the following results:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 20 | 40 | 80 | 160 |
| $n$ calculated | 105.2 | 85.5 | 57.9 | 26.9 | 7.0 | 0.8 |
| $n$ observed | 157.3 | 89.7 | 57.7 | 27.0 | 5.7 | 1.3 |
| Correction calc. | +52.1 | +4.2 | —0.2 | +0.1 | —1.3 | +0.5 |

The only notable deviation is found for intensity $h$ 10, which rains for the season 1889 were about 5 per cent more abundant than our formula demands.

201. The noted *Bergen coast* is represented in our table by the stations Bergen and Florœ. We find $a$ 2.198 and $b^1$ 0.0623, which give:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ ......... | 1 | 10 | 20 | 40 | 80 | 160 |
| $n$ calculated ........ | 157.8 | 136.8 | 104.7 | 61.7 | 24.3 | 5.5 |
| $n$ observed ......... | 167.5 | 130.0 | 105.0 | 61.5 | 24.5 | 5.0 |
| Correction of calc | +9.7 | —6.8 | +0.3 | —0.2 | +0.2 | —0.5 |

The agreement is astonishingly close except for the useful rains of first degree of intensity, which were less frequent by about 5 per cent than the calculated values. For the year 1889 these rains were as much high along the north coast as we have just seen (paragraph 200).

202. From the south coast of Norway we have the stations Mandal and Skudenes. We find $a$ 2.062 and $b^1$ 0.0655.

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ ......... | 1 | 10 | 20 | 40 | 80 | 160 |
| $n$ calculated ........... | 115.3 | 99.1 | 74.1 | 43.0 | 16.1 | 3.4 |
| $n$ observed ............ | 123.0 | 99.0 | 74.5 | 43.0 | 12.0 | 0.5 |
| Correction of calc... | +7.7 | —0.1 | +0.4 | 0.0 | —4.1 | —2.9 |

The heavy rains were less numerous than calculated.

203. For the entire southwest coast of Norway, from Mandal to Florœ, we find for the two preceding groups combined the values $a$ 2.140 and $c^1$ 0.0648, with the following result:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ ......... | 1 | 10 | 20 | 40 | 80 | 160 |
| $n$ calculated......... | 138.0 | 118.9 | 90.2 | 52.0 | 19.8 | 4.2 |
| $n$ observed.......... | 145.2 | 114.5 | 89.7 | 52.2 | 18.2 | 2.8 |
| Correction of calc.. | +7.2 | —4.4 | —0.5 | +0.2 | —1.6 | —1.4 |

The deficiency for $h$ 10 was noted in the Bergen region above. (See paragraph 201.)

204. The two stations, Christiana and Færder, on the east, or Bohus Bay, coast of Norway, are better represented by our formula (6) than by (7), as we should expect, since the great mountain chain is to the west of these stations. For (6), or with $x$ cube of log. $h$, we find $a$ 1.875 and $b$ 0.1955; and

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ ......... | 1 | 10 | 20 | 40 | 80 | 160 |
| $n$ calculated............. | 74.1 | 47.8 | 28.0 | 11.9 | 3.4 | 0.6 |
| $n$ observed............... | 92.0 | 47.5 | 28.5 | 12.0 | 5.0 | 1.5 |
| Correction of calc ..... | +17.9 | —0.3 | +0.5 | +0.1 | +1.6 | +0.9 |

The useful rains are very accurately represented. The heavy rains were in excess; we found them deficient in the preceding group, thus pointing to a seasonable peculiarity for the year 1889.

205. The data considered seem to be fully sufficient for our purpose. They most abundantly prove that our law, expressed in formula (6), correctly represents the results of observations made in the most diverse climatological regions of the globe. We found also that the

extreme case offered by the rainy coast of Norway is equally well expressed by our law, a slight variation in one of the constants representing the effect of the obstacle which the high mountain chain offers to the vapor laden sea winds.

206. In several instances the values calculated for the number of the highest intensities are rather high. If further research, extending over a series of years, should show this deviation to be real a term would have to be added to the formula (6), giving sensible values only for high rain intensities.

207. So far as apparent this term is parabolic, increasing with the square of the value of $(\log. h)^3$ diminished by 5, the limit below which this influence is entirely insignificant.

For the six stations of India the coefficient for this term is only three thousandths.

208. That is, we may apply the parabolic term

$$(9) \qquad \Delta\,(\log. n) = -\,0.003\,[(\log. h)^3 - 5]^2$$

to the calculated values for India and obtain results almost identical with observations even for the highest rainfall intensities. Applying this correction to the logarithm of $n$ calculated in paragraph 199, we obtain

| | | |
|---|---|---|
| $h$ | 100 | 200 |
| $n$ calculated | 15.0 | 5.0 |
| $n$ observed | 15.1 | 5.2 |
| Correction calculated | +0.1 | +0.2 |

209. At present there is no necessity for taking such additional term into consideration. If further research shall prove the reality of this term, it will represent an additional mechanism of nature tending to further diminish the frequency of rain of highest intensity below that given by the general law.

210. It will be interesting and quite instructive to bring the constants $a$ and $b$ for the different regions of the globe into a table to form the basis of some climatological comparisons.

211. It will be advisable to introduce some simple names for these characteristic quantities. Let us denote the total number of days with rainfall of a given intensity by adding the corresponding index number to $n$. Thus, the total number of rain days of the intensity 1 will be denoted by $n_1$, those of the intensity 4 by $n_4$, and the total number, including the insignificant rains, by $n_0$.

212. From the well known formula (6) we readily see that

$$(9) \qquad a = \log. n_0 \qquad\qquad a - b = \log. n_1$$

Or

$$(10) \qquad b = \log. n_0 - \log. n_1$$

The constant $a$ is the *logarithm of the total number* $n_0$ of rain days from $h = 1$ upwards, and $b$ is the *decrement* of the logarithm of the

number of rain days per unit of the cube of the logarithm of rain height.

213. It is evident that we may use either $a$ and $b$ or $n_0$ and $n_1$. In the latter case we must bear in mind that $n_0$ is not directly determined by observation, but has to be taken from the entire series of observations by calculation, as shown before (paragraph 120 *et seq.*). We shall also point out a theoretical reason for the fact that the registers of observation greatly overstate this value of $n_0$.

214. In the following table I give the two logarithmic values $a$ and $b$, and also the three numbers $n_0$, $n_1$, and $n_3$.

*Table of rain constants (calculated values).*

| Regions. | Logarithms. | | Numbers. | | |
|---|---|---|---|---|---|
| | $a$ | $b$ | $n_0$ | $n_1$ | $n_3$ |
| Iowa (twenty years) | 1.925 | 0.116 | 84.1 | 64.4 | 28.3 |
| India (six stations) | 1.892 | 0.086 | 78.0 | 64.0 | 29.0 |
| Lahore | 1.500 | 0.055 | 31.6 | 27.9 | 17.0 |
| Astrakhan | 1.55 | 0.32 | 35.5 | 17.0 | 1.7 |
| Russia (six stations) | 1.942 | 0.223 | 87.5 | 52.4 | 10.7 |
| Central Europe (two stations) | 2.078 | 0.201 | 119.7 | 75.3 | 18.1 |
| Holstein (four stations) | 2.129 | 0.202 | 134.6 | 84.5 | 20.1 |
| Norway— | | | | | |
| Bohus Bay (two stations) | 1.875 | 0.196 | 74.1 | 47.8 | 11.9 |
| Interior (two stations) | 1.82 | 0.24 | 66.1 | 38.0 | 6.9 |
| Norway coast— | | $c^1$ | | | |
| North coast (three stations) | 2.022 | 0.090 | 105.2 | 85.5 | 26.9 |
| Bergen coast (two stations) | 2.198 | 0.062 | 157.8 | 136.8 | 61.7 |
| South coast (two stations) | 2.062 | 0.066 | 115.3 | 99.1 | 43.0 |
| South Atlantic coast (preceding four stations) | 2.140 | 0.065 | 138.0 | 118.9 | 52.0 |

215. It will be understood, that either the logarithmic values $a$ and $b$, or *any* two out of the three numbers $n_0$, $n_1$, and $n_3$, fully determine the distribution of the rainfall according to its intensity in the particular region named.

216. For theoretical investigations the logarithmic constants will prove the most serviceable. They vary in so gradual and measured a manner that a new class of most general rainfall formulæ can be obtained by their means. As example, we might take the rainfall along the great continental axis from Holstein to Irkutsk.

217. For most climatological investigations a set of two numbers $n$ will best answer, and I strongly advise the adoption of $n_1$ and $n_3$ for this purpose, being the frequency of the lightest and heaviest agricultural rains, and both determinable directly with the highest degree of precision.

218. A complete new set of rain maps will thus be obtained. The most important, for practical purposes, will be the rain maps representing the frequency of the useful rains, determined by the values $n_1$ and $n_3$ (for $h = 10$ and 40 centi-inches or 2.5 and 10 millimeters). These maps should be supplemented by those for $n_4$ ($h$ 80 centi-inches or 20 millimeters).

219. While it will not be necessary it may be advisable to con-

tinue the maps of total rainfall amount. The rain frequency maps indicated will, however, give a fair idea of the total amount of rainfall, so that the maps representing the total rainfall will be of less importance hereafter.

220. This is hardly the place for extended climatological comparisons, based upon the new constants here presented. We will only call attention to the fact that $a$ and $n_0$ are smallest in the continental regions of Lahore and Astrakhan, and greatest on the Bergen coast. We also notice that the decrement $b$ is least for tropical and marine regions, and greatest for continental regions, averaging fully one-fifth for the continents of Asia and Europe, and attaining its highest value at Astrakhan, where it reaches one-third. In the Mississippi Valley this decrement is but little larger than in tropical India, and generally less than half the value characterizing Russia.

221. Before closing this chapter, we will, simply for the sake of completeness, transform the equation (6) representing our law, so as to more fully bring out the symmetry of the mathematical expression. For all practical purposes the original form (6) is the most convenient.

222. If we put

$$(12) \qquad \frac{b}{a} = \varepsilon^3$$

and introduce $a = \log. n_0$ from paragraph 212, the formula (6) is very readily transformed into

$$(13) \qquad \frac{\log. n}{\log. n_0} = 1 - (\log. h^{\varepsilon})^3$$

showing that the distribution may be represented as a cubic parabola of the logarithm $h^{\varepsilon}$.

## IV. THE LAW OF PROBABILITY.

223. The curves of Figs. 1 and 3, representing the total number of rain days of a given intensity in a given year for the most diverse climatic regions of the globe, will be recognized by many readers as possessing the characteristic properties of the curve of total probability up to a definite limit. In other words, our investigation has brought the relation of the frequency of rain and its intensity to the law of probability or of chances.

224. The law of probability has been quite thoroughly investigated, but mainly by methods of higher mathematics. Accordingly it is applied more frequently by astronomers than by other scientists, though the law of probability itself is of very great importance in most branches of science.

225. "The law of probability governs necessarily all phenomena of aggregate individuals and of repeated efforts, whether the individuals be the atoms of an inorganic body or the living and thinking men of

a nation; whether the effort be the coining of a five cent nickel or the determination of the right ascension of Sirius." (Hinrichs' School Laboratory, Vol. II, 1872; p. 38.)

226. In meteorology the law of probability will be of the highest importance. A considerable amount of work has already been done in this direction, but, on account of the lack of direct and handy methods, hardly more than a beginning has been made.

227. In the very meritorious recent work of Hugo Meyer,* of the Meteorological Institute of Prussia at Berlin, the importance of this law is clearly set forth, and numerous graphical applications thereof are made.

228. But even in this work we only find (page 18) the exponential equation of Gauss,

$$w_\Delta = w_0 e^{-\pi w_0^2 \frac{\Delta^2}{\varepsilon^2}}$$

which I copy in order to introduce the same to a wider public. We find also (p. 19) a fair drawing of the probability curve corresponding to this equation.

229. I will not stop to explain this Gaussian equation; it would take too much time, and do very little good. If we can not present the law of probability in a more simple and direct form, it will find but a limited field of usefulness in meteorological circles.

230. We hardly need add that even this author does not show how this complex equation is to be practically applied, and how the numerical values represented by this function are obtained. The equation is simply copied by him and exhibited as a sort of mathematical curiosity.

231. The most satisfactory exposition of this subject by the higher mathematics is given for the American public in Chauvenet's "Manual of Spherical and Practical Astronomy," 1863, Vol. II, pp. 478–486.

The same deduction may also be found in James C. Watson's "Theoretical Astronomy," Philadelphia, 1868, pp. 361–365.

232. We shall not enter upon this abstruse mathematical subject; it would be practically useless for our purposes. After completing our own exposition of the law of probability we shall return to these standard authorities on the mathematical side of the question simply to correct some of the results they have obtained and continue to repeat in standard examples. This will be done in order to satisfy the mathematical critic that the subject under investigation has been turned over not merely to secure practical utility but has also improved in clearness and in directness.

233. More than twenty years ago this subject presented itself to

* Anleitung zur Bearbeitung meteorologischer Beobachtungen für die Klimatologie. Berlin, 1891.

me in certain problems of molecular mechanics or theoretical chemistry. I took up the investigation in an entirely elementary manner, applying in its solution only the general method of quantitative induction, as set forth in my little treatise on that method published in 1872. Some of my practical results on the establishment and application of the law of probability were published in my "School Laboratory of Physical Science," Vol. II, Iowa City, 1872, pp. 28–38. As that exposition is incomplete and not very accessible, I deem it best to briefly give an independent exposition of this investigation here, whereby the method of applying the same practically will be most readily understood.

234. Discarding all theory and all mathematical philosophizing, I set about obtaining experimental data and then, by the method of induction, tried to obtain the general law, as I would have done in any other series of data of experiment or observation. In fact, I completely ignored the mathematical investigations made, and took up the problem as entirely new and exclusively as a matter of fact.

235. From a large stock of white and red marbles I selected carefully a number that were most nearly equal in size and weight. I soon had to repeat this sifting with much greater care, for I found quite appreciable constant differences in the results; and, finally, had to be satisfied with balls that were not perfectly equal, as my experiments show.

236. I obtained a wooden urn, rather narrow and quite deep—fully 12 inches deep and barely wide enough to freely stir the balls therein with one hand.

237. Into this urn I put a definite number of the white and red marbles (equal in number or not) and had one of my students stir the balls thoroughly and draw a *definite* number; take them out, count the number, $r$, of *red* balls in the draw, and return the balls to the urn for the next trial. Another student recorded the number so drawn.

238. By entering this number on quadrilated paper of fields marked ten by ten, it was easy to make series of experiments of a hundred draws in each. Usually each set of two students had to make ten such series of experiments, or a thousand draws each.

Each series of one hundred draws was summed up separately, so as to show exactly how much the consecutive series varied.

239. Early in the seventies I had made in this way not less than half a million of draws under the most varied conditions and by different students. I will here only give the most necessary specimens of the simplest kinds to obtain the actual data of experiment from which to deduce the general law of probability in a form directly and easily applicable to meteorological investigations, and involving elementary mathematics only.

240. From the record of Mr. Charles D. Ramsdell, one of my students,* I copy the following *first series* of hundred draws; the number given is the number of *red* balls in each draw of six balls from an urn containing ten white and ten red balls, all as nearly equal in weight and size as possible:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 3 | 2 | 3 | 2 | 3 | 5 | 4 | 2 | 4 |
| 4 | 3 | 3 | 5 | 3 | 3 | 3 | 3 | 2 | 3 |
| 3 | 4 | 4 | 4 | 4 | 2 | 4 | 3 | 5 | 3 |
| 2 | 3 | 2 | 3 | 4 | 2 | 2 | 3 | 3 | 4 |
| 3 | 4 | 2 | 2 | 3 | 1 | 6 | 1 | 2 | 1 |
| 4 | 3 | 3 | 3 | 1 | 3 | 2 | 4 | 3 | 1 |
| 2 | 3 | 2 | 4 | 4 | 1 | 1 | 3 | 1 | 3 |
| 3 | 3 | 4 | 3 | 4 | 1 | 3 | 4 | 2 | 3 |
| 3 | 3 | 3 | 1 | 2 | 4 | 3 | 5 | 4 | 1 |
| 3 | 3 | 3 | 1 | 2 | 4 | 3 | 3 | 2 | 2 |

241. Summing up these draws, we find the following as the number of times that each given set of balls has been drawn in this first series, namely:

| No. of red balls drawn. | No. of times drawn. |
|---|---|
| None | 0 |
| One | 12 |
| Two | 20 |
| Three | 42 |
| Four | 21 |
| Five | 4 |
| Six | 1 |
| Total draws made | 100 |

242. The following table gives the result for each of the ten series of draws made by the same student (Ramsdell), together with the *mean*, *P*, of the series, exactly as he has left the record:

| Red balls, *r*. | Series. | | | | | | | | | | Mean, *P*. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | I. | II. | III. | IV. | V. | VI. | VII. | VIII. | IX. | X. | |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 2 | 0 | 0 | .5 |
| 1 | 12 | 11 | 5 | 8 | 5 | 1 | 7 | 4 | 4 | 4 | 6.1 |
| 2 | 20 | 34 | 29 | 21 | 20 | 22 | 27 | 20 | 24 | 18 | 23.5 |
| 3 | 42 | 33 | 32 | 38 | 41 | 38 | 34 | 38 | 40 | 40 | 37.6 |
| 4 | 21 | 26 | 25 | 25 | 30 | 28 | 25 | 24 | 25 | 30 | 25.9 |
| 5 | 4 | 6 | 6 | 8 | 2 | 4 | 6 | 11 | 7 | 8 | 6.2 |
| 6 | 1 | 0 | 2 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | .6 |

243. This table—only one of a great multitude—might be made the subject of some very curious remarks, but it would not be directly to the main point in question. All I will say here is, that Mr. Ramsdell was a very painstaking and faithful student, whose only aim was to do every kind of work in the best manner he was capable of. From

* During the school year 1871–1872 two hundred and seventy-two students worked in my physical laboratory from two to ten hours each week.

the teacher's mark on his paper I conclude that he spent four entire afternoons on these one thousand draws.

244. The mean, marked $P$, expresses, evidently, the mean chance in a hundred draws of drawing the given number of red balls in draws of six from an urn containing an equal number of red and white balls. Or, if we disregard the decimal point, the last column gives the number of times that each given number of red balls actually was drawn in a thousand draws.

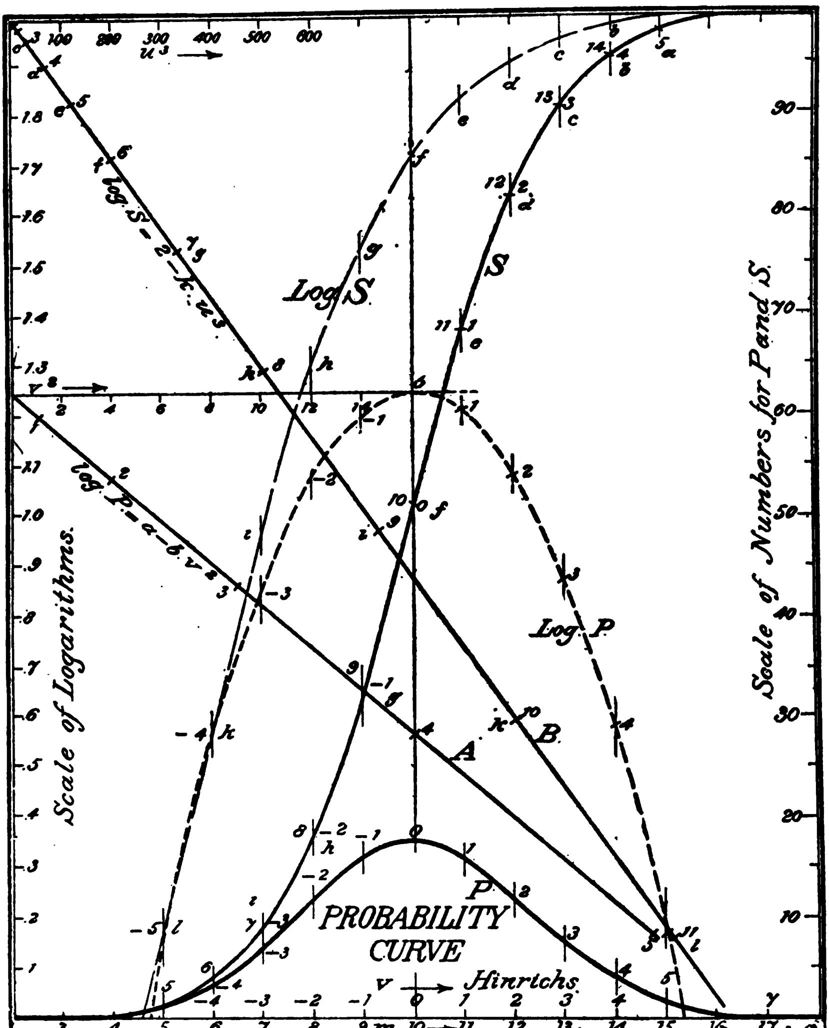


FIG. 5.

Thus, three hundred and seventy-six times three red and three white balls were drawn in a thousand trials, and only five times were all six balls white, and six times were they all red.

245. Now this number $P$ is evidently the *probability* in a hundred chances of drawing the given number of balls. If, therefore, on a basis of equal parts we set off verticals equal to these means and join the seven points so determined, the resulting curve will be the *curve of probability* for the given set of experiments made. The general appearance of this curve will be like the one marked $P$ in Fig. 5.

246. Mere inspection shows that this curve is not the representation of an algebraic function but of a transcendental one. Compare my "Method of Quantitative Induction" (12, 13, and 76 to 84).

It is, therefore, indicated next to plot the logarithm of the probability $P$ over the same values of $r$. Upon actually doing so we obtain a curve like the dotted one marked log. $P$ in Fig. 5.

247. By the method of quantitative induction this curve is very readily recognized as a common *parabola*, for which we have

$$\text{(15)} \qquad \log. P = a - b\,v^2$$

where $v$ represents the abscissa counted from the middle line, $r = 3$, to the right or left.

248. The actual quantitative *test* of the correctness of this conclusion we obtain by constructing the points $y = \log. P$, $x = v^2$, and by means of a true straight ruler finding whether they are located in a straight line or not.

$$\text{(16)} \qquad y = a - b\,x$$

For the parabola (15) evidently is reduced to the straight line (16) by the substitution specified above.

In practice the ordinary $y = \log. P$ remains unchanged, and for the new abscissæ, $x = v^2$, it is easy to adopt a convenient scale, as exemplified in Fig. 5, line $A$.

249. Let us test the thousand draws of Mr. Ramsdell, the results of which are tabulated in paragraph 242.

In the first place, we notice that the values are not exactly symmetric, being a little higher for the higher values of $r$, the number of red balls. This is due to a slight difference between the average size and weight of the red balls as compared to the white balls, which I stated (paragraph 235) it was found impossible to remove entirely at the time.

250. The only thing we can do now will be to take the mean of the values for the same $v$. Thus we obtain—

Probability found

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$.................. | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $v$.................. | —3 | —2 | —1 | 0 | 1 | 2 | 3 |
| Probabilities ... | 0.5 | 6.1 | 23.5 | 37.6 | 25.9 | 6.2 | 0.6 |

Means for same numerical value of $v$:

| | | | | | |
|---|---|---|---|---|---|
| $v$ | = | 0 | 1 | 2 | 3 |
| $P$ | = | 37.6 | 24.7 | 6.15 | 0.55 |
| Log. $P$ | = | 1.575 | 1.393 | 0.789 | 0.740–1 |
| $v^2$ | = | 0 | 1 | 4 | 9 |

251. By applying the ruler we find that the straight line passing through the points for $v$ 0 and 2 will deviate the least from all points determined. The corresponding drawing (or in this case simple calculation) gives for the value of the constants in (16): $a$ 1.575 and $b$ 0.1965.

252. By means of these constants the equation (15) allows us to calculate the probability of each draw. We find for

| | | | | |
|---|---|---|---|---|
| $v$ | 0 | 1 | 2 | 3 |
| $P$ calculated | 37.6 | 23.9 | 6.15 | 0.64 |
| $P$ observed | 37.6 | 24.7 | 6.15 | 0.55 |
| Correction of calculation | 0.0 | +0.8 | 0.0 | −0.09 |

253. This is a most satisfactory agreement. We can, therefore, assert it is a fact that the logarithm of the probability forms a parabola over the base of chances (see paragraph 247), and that the probability can be calculated by (15), the constants having been determined graphically according to (16).

254. Since there is an apparent difficulty in reducing the case of drawing an odd number of balls, I will add the results obtained by Mr. A. H. Hull in ten series of one hundred draws each, drawing five balls every time. The mean of his ten series is as follows:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$ | = | 0 | 1 | 2 | 3 | 4 | 5 |
| $v$ | = | $-\frac{5}{2}$ | $-\frac{3}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{3}{2}$ | $\frac{5}{2}$ |
| $P$ | = | 1.8 | 13.4 | 34.8 | 34.8 | 13.4 | 1.8 |

The constants of (16) are $a$ 1.5934 and $b$ 0.2072, with the following results:

| | | | |
|---|---|---|---|
| $v$ | $\frac{1}{2}$ | $\frac{3}{2}$ | $\frac{5}{2}$ |
| $P$ calculated | 34.8 | 13.4 | 1.9 |
| $P$ observed | 34.8 | 13.4 | 1.8 |
| Correction calculated | 0.0 | 0.0 | −0.1 |

255. In order to obtain a definite idea how far such experiments agree, when made by different persons at different times, I will add the results obtained by ten series of a hundred six-ball draws, each made by Miss A. R. Tanner:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $P$ | 0.4 | 5.5 | 25.3 | 37.4 | 24.6 | 7.2 | 0.5 |

By taking the mean for corresponding numbers and comparing the same with the observations of Ramsdell (see paragraph 250), we obtain:

| | | | | |
|---|---|---|---|---|
| $v$ | 0 | 1 | 2 | 3 |
| $P$ (Tanner) | 37.4 | 24.95 | 6.35 | 0.45 |
| $P$ (Ramsdell) | 37.6 | 24.7 | 6.15 | 0.55 |
| Mean | 37.5 | 24.82 | 6.25 | 0.50 |

The agreement is remarkably close.

256. Constructing the curves and determining the constants as shown before, we obtain for the probability of these 2,000 draws by

4

two distinct sets of observers, the constants $a$ 1.574 and $b$ 0.195, with the following results:

| | | | | |
|---|---|---|---|---|
| $v$ | 0 | 1 | 2 | 3 |
| $P$ calculated | 37.5 | 23.7 | 6.25 | 0.66 |
| $P$ observed | 37.5 | 24.8 | 6.25 | 0.50 |
| Correction of calculation | 0.0 | +1.1 | 0.00 | −0.16 |

257. There can remain not the slightest doubt about the correctness of our formula of the probability curve; it is the parabolic (15) of paragraph 247.

$$\text{(15)} \qquad \log. P = a - b\, v^2$$

We have shown how we may determine the constants $a$ and $b$ by graphical method. The application of the law of probability can therefore now be made without any difficulty whatever. For application in chemistry see my "Principles of Chemistry and Molecular Mechanics," Davenport and New York, 1874, pp. 135–137.

258. In closing this subject, we may state that the formula (15), paragraph 247, which we found by our method of quantitative induction, not only expresses the observed facts, but really is identical with the form in which the theoretical equation first appears at integration, see for example, Chauvenet, Vol. II, p. 483, second equation from bottom of the page. In fact, there is a curious leap in the theoretical demonstration when (on p. 482) the logarithmic function is introduced.

259. It might yet be objected that our graphical determination of the constants $a$ and $b$ is inferior to the determination of these constants by the method of least squares. I have considered the corresponding case for the total probabilities in paragraph 127.

I may be permitted to give a very forcible, practical illustration of this point. (See paragraph 232.)

260. Thinking that I would obtain an excellent illustrative example for use by taking the case given by Chauvenet (Vol. II, p. 489), I plotted one column of the values tabulated (p. 490), supposing I had taken the observed data. To my utter astonishment I found the law inapplicable under one set; the series of ten data was most distinctly divided into two groups, representing two distinct straight lines (compare paragraph 248) of which the one gave the constants $a$ 2.032 and $b$ 0.0133, and represented the points one to four, while the other gave $a$ 2.005 and $b$ 0.0124, and represented the points from five to ten.

In astonishment at this strange result, I naturally turned again to the data used. I discovered that I had made a mistake. I had taken the theoretical values, calculated by Bessel himself, and taken from his "Fundamenta Astronomiæ" by Chauvenet (Vol. II, p. 489). The data represent the errors in right ascension of Sirius, according to the observations of Bradley.

261. Repeating my work for the actual data of experience, I had

no difficulty in applying my method, and obtained the constants $a$ 1.995 and $b$ 0.0122.

This shows conclusively, that the theoretical reductions made by no less authority than Bessel, himself, by using elaborate and very troublesome calculations according to the method of least squares, gave final results so much at fault as to show a marked break in the series of results, which it was impossible to represent by a single straight line in my graphic method.

I should like to give copy of my graphics of this notable case, but space forbids. In order to present all the necessary data, I will, however, give the values *observed* (headed, by Chauvenet: No. of errors by experience), and the values calculated from law of probability by Bessel (headed: No. of errors by theory, in Chauvenet) I will add the values calculated by me from the constants which I obtained by my graphical method. The unit of error in right ascension is one-tenth of a second; it is the unit of $v$.

| $v$ | Probability. | | | Deviations. | |
|---|---|---|---|---|---|
| | Observed. | Calculated. | | | |
| | | Bessel. | Hinrichs. | Bessel. | Hinrichs. |
| 1 | 94 | 95 | 96.2 | − 1 | − 2.2 |
| 2 | 88 | 89 | 88.3 | − 1 | − 0.3 |
| 3 | 78 | 78 | 76.7 | 0 | + 1.3 |
| 4 | 58 | 64 | 63.1 | − 6 | − 5.1 |
| 5 | 51 | 50 | 49.0 | + 1 | + 2.0 |
| 6 | 36 | 36 | 36.0 | 0 | 0.0 |
| 7 | 26 | 24 | 25.0 | + 2 | + 1.0 |
| 8 | 14 | 15 | 16.4 | − 1 | − 2.4 |
| 9 | 10 | 9 | 10.2 | + 1 | − 0.2 |
| 10 | 7 | 5 | 6.0 | + 2 | + 1.0 |

262. Taking the sum of the squares of the deviation the column of Bessel gives 49, and my column only 44.43.

That is, the result calculated by a laborious application of the method of "least" squares gives a sum of squares which is 10 per cent in excess of the sum of squares for the values which were obtained by direct graphical process.

263. I disliked greatly to introduce this case, because it may be misconstrued. But the prevailing tendency of the dominant school of scientists is to disregard graphical methods as inferior. This prevailing opinion I know to be erroneous. I have, therefore, deemed it proper to demonstrate my opinion by examining a case that is classical, due to the very masters, and given as a model. Now, this very model is so badly out of gear that its values are in conflict with the law of probability by which they were professedly obtained by the most refined method of calculation.

These values were instantly recognized to be in that deplorable condition as soon as I put my ruler on the drawing, attempting to

draw a single straight line coming reasonably near to all the points determined.

264. The point here made in favor of the use of the graphical method is of the utmost practical consequence to the progress of scientific meteorology. If, under a mistaken estimate of the value of the methods before us, we should deem the elaborate method of the least squares a necessity, we would continue to find very little solid work done in this line; it would not be possible to accomplish it.

265. But if meteorologists become convinced of the fact that our graphical method* is fully as accurate in its results, and incomparably easy in application, they will make abundant use of it which will of necessity greatly increase the results of general meteorology.

266. There is still another most weighty reason in favor of the graphic method, for by this method it is practically impossible to commit such stupendous errors as have been committed by the calculatory process of the method of least squares. It would be unpleasant to specify cases unless compelled to do so.

267. Having established the simple mathematical expression of the probability, we will next apply the same method of quantitative induction to the determination of the *sum of probabilities* or the *total probability* up to any given value of the independent.

268. In meteorology this branch of the subject is really the most important. We generally can determine, by observation, the total probability more correctly than the simple probability for a given value. It is evident that any error affecting a given point will not be carried on to the next, but really is eliminated by the process of summation.

269. Proceeding as before, let us make use of the series of one thousand draws, reported in paragraph 242, taking the values equalized for error in equality of the balls, as stated in paragraph 250.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$........ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $P$........ | 0.55 | 6.15 | 24.7 | 37.6 | 24.7 | 6.15 | 0.55 |
| $S$........ | 0.55 | 6.70 | 31.4 | 69.0 | 93.7 | 99.85 | 100.40 |

270. We see that the sum of all probabilities or chances of drawing six balls containing no red ball is 0.55, of drawing up to one red ball is 6.70, of drawing up to two red balls is 31.4 in a hundred.

We also notice that the record shows an error of four draws in the thousand. Being insignificant, we will let it stand.

271. Plotting the points so determined, our $r$ as abscissa, and $S$, the sum of probabilities $P$, as ordinates, we obtain a curve like $S$ in Fig. 5.

* Drawings must, of course, be carefully made on properly large scale. Quadrilated co-ordinate papers are fit for sketches only.

This curve is immediately recognized as identical in form with those we have found representing the total rainfall frequency.

272. Proceeding in regular order of induction, exactly as if we had found this curve for the first time, we would, as in paragraph 246, recognize that it is *not* the expression of an algebraic function, nor a period function; hence pass to the logarithmic at once.

273. To observed numbers of $S$ given above belong the following logarithms:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$......... = | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $S$........ = | 0.55 | 6.70 | 31.4 | 69.0 | 93.7 | 99.85 | 100.40 |
| Log. $S$. = | —0.260 | 0.826 | 1.497 | 1.839 | 1.972 | 1.999 | 2.002 |

Plotting these points, we find they determine a parabolic curve like the one marked log. $S$ in Fig. 5.

274. To determine the exponent of $r$, we obtain on plotting the same log. $S$ over $r^3$ again a parabolic curve of but a slight curvature. Trying $r^4$ we obtain a straight line, consequently the exponent is 4, and the unknown function is

$$(17) \qquad \log. S = a - b\, r^4$$

275. Carrying out the construction on a sufficiently large scale and with precision, we find for the values of the constants $a$ 2.002 and $b$ 0.00188. The following table shows how closely the values of $S$ calculated by means of these constants from (17) reproduce the observed values $S$:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$........................ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $S$ calculated......... | 0.70 | 5.8 | 31.2 | 69.5 | 93.4 | 100.0 | 100.4 |
| $S$ observed........... | 0.55 | 6.7 | 31.4 | 69.0 | 93.7 | 99.9 | 100.4 |
| Correction calc..... | —0.15 | +0.9 | +0.2 | —0.5 | +0.3 | —0.1 | 0.0 |

The agreement is perfectly satisfactory, the deviations small and both positive and negative.

276. It will be noticed that we have here considered the total number of chances up to and including the number $r$ of red balls drawn. Hence, for $r = 3$ we find 69 and not 50 as the result. This produces the same kind of asymmetry which we have noticed in the case of the total rain frequency on the west coast of Norway.

277. We ought to repeat this summation in the more common manner, taking the probability $P$ as belonging to the point $r$ in the curve $P$ and reaching midway to the preceding and following point. That is, taking $S$ as the area of the curve $P$.

To avoid mere repetition we prefer to apply our method to the theoretical data instead of the experimental draws thus far made use of.

278. It is well known that the so-called *binomial coefficients* of $\frac{1}{2} + \frac{1}{2}$ raised to a given power represent the chances of drawing a number of balls equal to that power from an urn containing an equal

number of red and white balls. For drawing twenty balls the chances of probability $P$ of drawing $r$ 0, 1, 2 to 20 red balls and the total probability or sum $S$ are as follows, per one hundred draws:

| r. | v. | P. | S. | Log P. | Log. S. |
|---|---|---|---|---|---|
| 0 | 10 | 0.00 | 0.00 | .......... | .......... |
| 1 | 9 | 0.01 | 0.01 | .......... | .......... |
| 2 | 8 | 0.02 | 0.06 | .......... | .......... |
| 3 | 7 | 0.11 | 0.11 | —0.959 | —0.959 |
| 4 | 6 | 0.46 | 0.40 | —0.337 | —0.398 |
| 5 | 5 | 1.45 | 1.35 | 0.161 | 0.136 |
| 6 | 4 | 3.70 | 3.93 | 0.568 | 0.594 |
| 7 | 3 | 7.39 | 9.48 | 0.869 | 0.977 |
| 8 | 2 | 12.02 | 19.13 | 1.080 | 1.282 |
| 9 | 1 | 16.03 | 33.16 | 1.205 | 1.520 |
| 10 | 0 | 17.66 | 50.00 | 1.247 | 1.699 |
| 11 | — 1 | 16.03 | 66.84 | 1.205 | 1.825 |
| 12 | — 2 | 12.02 | 80.87 | 1.080 | 1.908 |
| 13 | — 3 | 7.39 | 90.53 | 0.869 | 1.957 |
| 14 | — 4 | 3.70 | 96.07 | 0.568 | 1.983 |
| 15 | — 5 | 1.45 | 98.65 | 0.161 | 1.994 |
| 16 | — 6 | 0.46 | 99.60 | —0.337 | 1.999 |
| 17 | — 7 | 0.11 | 99.89 | —0.959 | 1.999 |
| 18 | — 8 | 0.02 | 99.94 | .......... | 1.999 |
| 19 | — 9 | 0.01 | 99.99 | .......... | 1.999 |
| 20 | —10 | 0.00 | 100.00 | .......... | 2.000 |

279. These values are represented in Fig. 5, in the curves marked as follows:

$P$, the probability curve.

$S$, the curve of total probabilities, or the sums of $P$.

Log. $P$, the curve of the logarithm of probability, and

Log. $S$, the curve of the logarithm of total probabilities.

280. The first striking feature consists in the very gradual diminution of the values towards $r = 0$ and $r = 20$. Hence the drawing begins at $r = 2$ and ends at $r = 18$. In other words, the most important portions of all curves are the more central portions for $v = 0$, to 6 or 7.

281. Mere inspection shows that the curve log. $P$ is probably a common parabola. Constructing log. $P$ on the new abscissa $x = v^2$ tangent to the vertex, we obtain the straight line $A$, thus demonstrating log. $P$ to be really the common parabola.

$$\text{(15)} \qquad \log. P = a - b\, v^2$$

282. The drawing shows that $a$ 1.245 and $b$ 0.0426. Calculation gives the following values of $P$:

| $v$ ............ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P$ calculated | 17.66 | 16.03 | 11.89 | 7.31 | 3.67 | 1.52 | 0.52 | 0.18 |
| $P$ observed.. | 17.66 | 16.03 | 12.02 | 7.39 | 3.70 | 1.45 | 0.46 | 0.11 |
| Cor. calc..... | 0.00 | 0.00 | +0.13 | +0.08 | +0.03 | —0.07 | —0.06 | —0.06 |

The "observed values" are the theoretical probability expressed by the binomial coefficients. The values calculated by my formula are identical therewith.

283. The parabolic curve, log. $S$, offers one difficulty, namely, an

indefinite beginning.* Evidently, we may take the point marked $a$ as the first, $b$ as the second, and so on.

In other words, instead of beginning with $r = 0, 1, 2$, we begin with $u = 0$ and the point of $S$ for which $r = 15$, we count $u = 1$, where $r = 14$, we count $u = 2$, and so on. With the scale marked at the top, we can now determine the points log. $S$ and $u^3$. We find they fall in the straight line marked $B$. Hence we have

$$\log. S = a - b\, u^3$$

284. The drawing gives the constants, $a$ 2.000 and $b$ 0.001406, with the following result:

| | | | | | | |
|---|---|---|---|---|---|---|
| $r$ .................... | | 15 | 14 | 13 | 12 | 11 |
| $u$ .................... | 0 | 1 | 2 | 3 | 4 | 5 |
| $S$ calculated......... | 100. 0 | 99. 8 | 97. 5 | 91. 6 | 81. 3 | 66. 7 |
| $S$ observed........... | 100. 0 | 98. 7 | 96. 1 | 90. 5 | 80. 9 | 66. 8 |
| Correction of calc.. | 0. 0 | —0. 9 | —1. 4 | —1. 1 | —0. 4 | +0. 1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$ ................ | 10 | 9 | 8 | 7 | 6 | 5 | 4 |
| $u$ ................ | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| $S$ calculated..... | 49. 7 | 33. 0 | 19. 1 | 9. 4 | 3. 9 | 1. 4 | 0. 3 |
| $S$ observed...... | 50. 0 | 33. 2 | 19. 1 | 9. 5 | 3. 9 | 1. 4 | 0. 3 |
| Cor. of calc...... | +0. 3 | +0. 2 | 0. 0 | +0. 1 | 0. 0 | 0. 0 | +0. 1 |

285. The agreement between calculation and observation is perfectly satisfactory here also, though there are deviations up to one per cent at the beginning, deviations due to starting in at the point chosen as sufficiently near the perceptible departure of the curve $S$ from the horizontal.

286. It may yet be said that while it is very easily proved that the probability curve and binomial curve must be identical, the binomial coefficients are not matters of experience, but of mathematical theory.

In my very first scientific memoir I showed how these so-called binomial coefficients of Newton can be obtained without making any hypothesis whatever. (See "Beviis for Newton's Bi-nomial Formel" in Matematisk Tidsskrift, Kjœbenhavn, 1860, pp. 38–42.)

287. We therefore may indeed consider the binomial curve as correctly the probability curve. And since our simple formulæ very satisfactorily reproduce the binomial values we have herein an independent confirmation of the correctness of our formulæ.

288. Our formulæ thus rest both upon the actual data of experiment obtained by *de facto* drawing balls from an urn and also upon the most direct, simple mathematical consideration of the chances expressed in the binomial coefficients, which latter I have long ago shown to be independent of any hypothesis whatever.

289. We have also seen that if we count up to the full draw the

* It may be noticed that this fact also affects the constant deviation for $h = 1$ in the total rain frequency.

curve of probability is asymmetric and represented by the fourth power; but if we count in the ordinary manner, so that the middle of $S$ is marked by half the total number, we have a symmetrical curve of probability, and the total probability is represented by the third power.

290. Having divested the law of probability of all theoretical difficulties, established the same as an experimental fact, and expressed it by simple formulæ obtained by the easiest and most direct method of quantitative induction, it may be reasonably expected that the law will soon find that prominence in scientific meteorology to which it is entitled, since its application hereafter will not only be most simple and direct but also divested of all unnecessary labor of calculation by making the graphical method the basis of the process.

## V.—THE LAW OF RAIN PROBABILITY.

291. We have found in Chapter IV exactly the same mathematical expression for the total probability as for the total rain frequency in Chapter III. It therefore demonstrates that the rain frequency is determined by the law of probability.

While the general law is there fully established, it may be advisable to enter into some of the more important particulars of this interesting subject.

292. The curves in Fig. 3 represent the total rain frequencies for the different regions, as function of rain intensity. If we subtract each ordinate from the preceding, the difference will represent the number of rains for the interval of intensity, and will thus measure the rain probability of that intensity.

293. In this manner a new set of curves, not here given, have been obtained. By measuring the ordinates, we find the following numbers:

*Rain probability or number of rain days in* 1889.

| | Insignificant rains. | | | Useful rains. | | | Damaging rains. | |
|---|---|---|---|---|---|---|---|---|
| | $h$ (centi-inches). | | | | | | | |
| | 1¼ | 1½ | 5 | 10 | 20 | 40 | 80 | 160 |
| India | 5.8 | 9.8 | 13.0 | 15.2 | 16.0 | 15.5 | 12.3 | 7.5 |
| Iowa* | 9.0 | 14.4 | 19.2 | 21.0 | 19.5 | 15.4 | 9.4 | 4.0 |
| Russia | 17.3 | 25.0 | 27.7 | 24.2 | 16.6 | 10.3 | 2.4 | ........ |
| Holstein | 8.8 | 24.3 | 33.4 | 36.2 | 28.8 | 16.0 | 4.5 | ........ |
| Norway† | 7.0 | 11.2 | 18.3 | 29.1 | 39.0 | 37.2 | 24.4 | 5.0 |

*Iowa City, ten years, 1881–1890. †Bergen coast.

The first three values are less certain, no counting of the original registers having been made between 1 and 10 centi-inches.

294. In order to bring out the form of these probability curves more strikingly, I have reconstructed the same in Fig. 6, using a smaller scale for the intensity base.

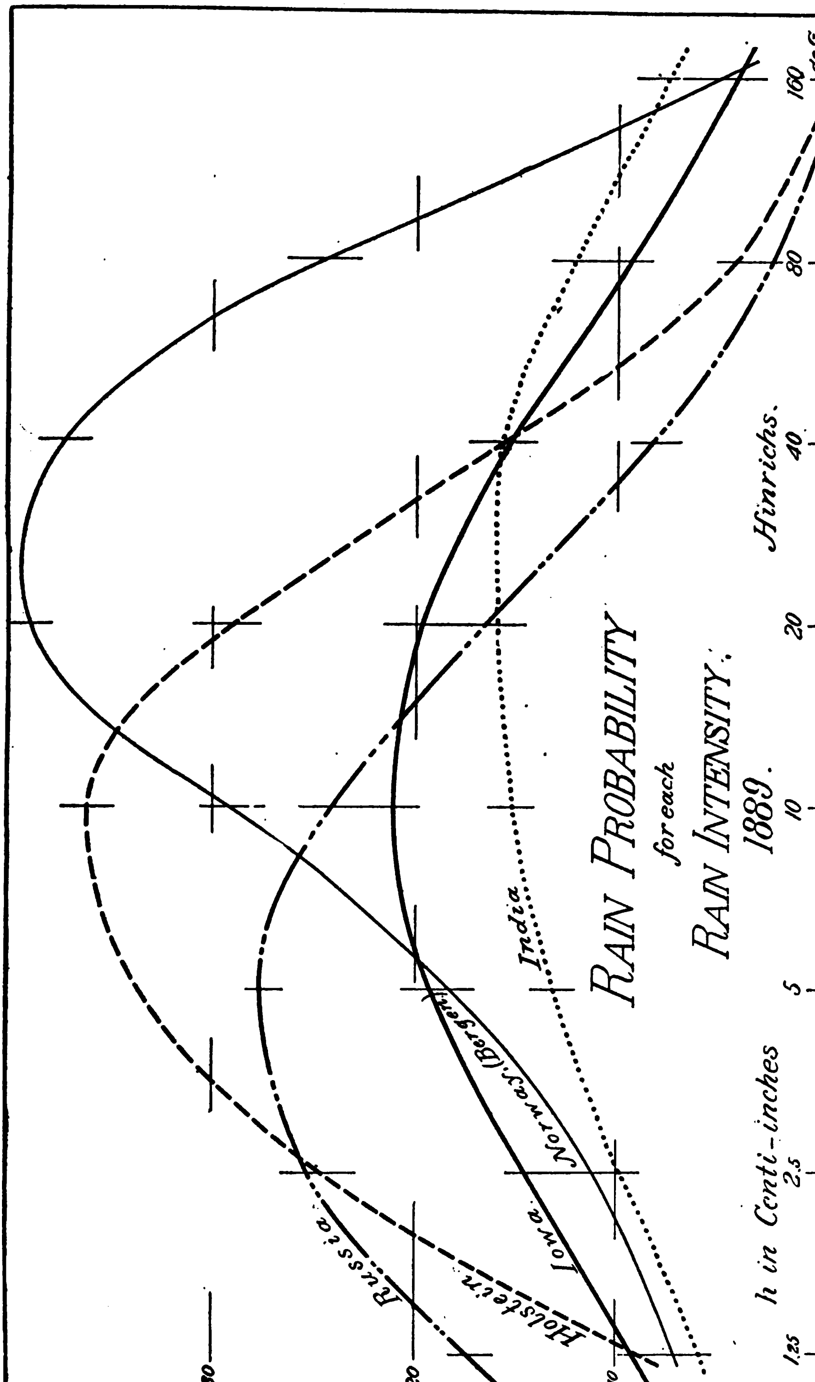
RAIN PROBABILITY
for each
RAIN INTENSITY.
1889.
Hinrichs.
h in Centi-inches
1
1.25
2.5
5
10
20
40
80
160
10
20
30
India
Norway (Bergen)
Iowa
Holstein
Russia

Fig. 6.

All of these curves have clearly the characteristic form of the probability curve marked $P$ in Fig. 5.

295. The curves for Iowa and for India are the most symmetrical. The highest rain probability of Iowa is about twenty-one a year of 0.10 inch. Both toward higher and lower intensity the rain probability diminishes, remaining as high as four a year for the days rainfall of 1.60 inch.

296. The rain probability curve for India reaches its highest point (16) at a much higher rainfall, namely at 40 centi-inches. The most frequent rain days in India bring therefore four times as much water each as do the most frequent rain days in Iowa. For lower intensities, the probability is less, for higher intensities it is greater in India than in Iowa.

297. The rain probability curve for Holstein (on land and sea) has its highest point (36) for the same rain intensity of 10 centi-inches as in Iowa. The lower intensities are much more frequent in Holstein than in Iowa, while the number of rains of higher intensity falls off much more rapidly in Holstein than in Iowa, so much so, that rains of 160 centi-inches have no appreciable probability in Holstein. Iowa has less light rains interfering with farm work, but more heavy and washing rains, than Holstein.

298. The probability curve for Russia attains its highest point, 28 a year, for the insignificant rains of the intensity of about half a unit or 0.05 inch; for higher intensities, the rain frequency in Russia drops off as rapidly as in Holstein, the actual probability remaining constantly numerically much lower in the continental empire than in the marine province of Holstein.

299. For Norway, only the probability curve of the Bergen coast has been drawn in Fig. 6. It is the most asymmetric of all. The insignificant rains have only the probability of India, but rains of 20 centi-inches occur about forty times a year, while rains of higher intensity on the Bergen coast remain even more probable than in India, until the intensity of almost 160 centi-inches is reached, after which the probability in India remains in excess of that in Norway.

300. These curves are exceedingly interesting. All but that for Iowa have been determined by the results of one single year of observations only (1889); this has been possible because we had discovered the true intensity scale.

The fact that the frequency curves of Fig. 6 have the form of the probability curve thus demonstrates the correctness of our rain intensity scale.

## VI.—THE LAW OF TOTAL RAINFALL.

301. When rainfall observations are discussed according to the method here set forth, which is based upon our rational rain-intensity

scale, the total amount of rainfall is of secondary importance, because any two rain frequencies for definite intensities give a very satisfactory representation of the rain distribution. As such frequencies, I have shown that those for $h = 10$ and 40 centi-inches are the most suitable.

302. It will readily be understood on simple mathematical principles that my rational rainfall frequency curves really implicitly give also a perfectly sufficient representation of the total rainfall amount.* Still I deem it very useful also to take up the subject of total rainfall independently by the same method that has proved itself so valuable in the study of rain frequency.

303. I shall, accordingly, take up the study of total rainfall on the basis of my own twenty years' observations, and arrange them according to my rain intensity scale, here temporarily again taking the full inch scale, for the reasons fully explained. (Paragraphs 79 to 85.)

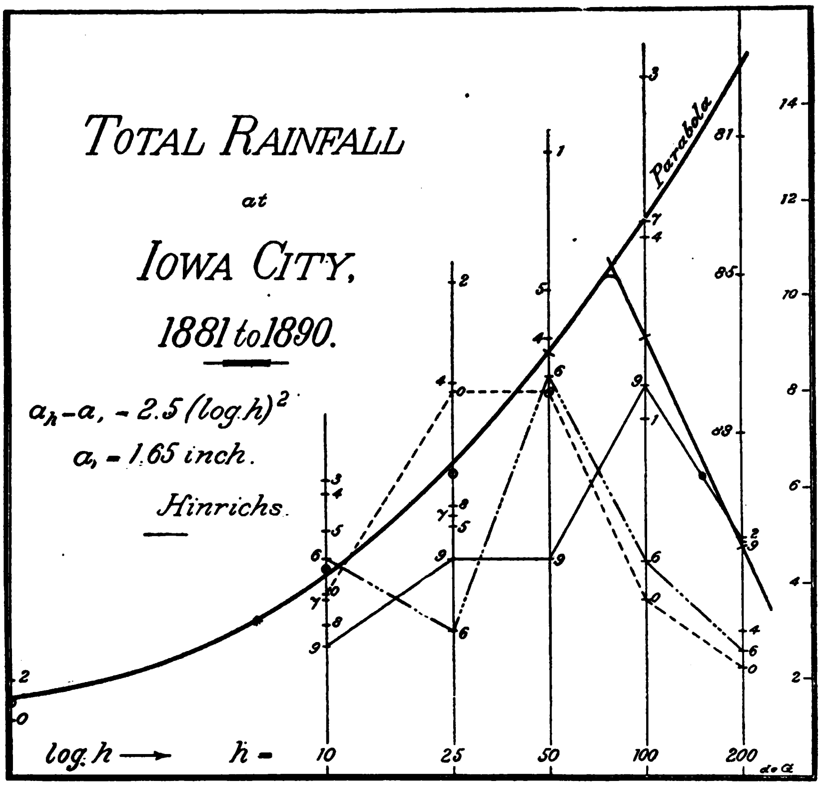


FIG. 7.

304. My ten years' series, from 1881 to 1890, gives the following

* The mean daily rainfall for each intensity being known. (See paragraph 489.)

total amount, $a_h$, in inches for each rain height, $h$, of this preliminary intensity scale:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ (centi-inches) | 1 | 10 | 25 | 50 | 100 | 200 |
| $a_h$ (inches)...... | 1.57 | 4.31 | 6.38 | 8.88 | 9.17 | 4.87 |

305. These observed values have been plotted in Fig. 7. The original drawing represented the rain depth in full scale, that is, one inch rainfall was represented by a linear inch.

Joining the points graphically, it is seen that two distinct geometrical forms are produced, namely, a rising *parabolic curve* for the insignificant and useful rains and falling *straight line* for the damaging rains.

306. Applying the method of quantitative induction the curvilinear part is found to be the common parabola,

$$(19) \qquad y = K x^2$$

where

$$y = a_h - a_1$$

and $K = 2.5$ inches, $a_1 = 1.65$ inches, and according to construction $x = \log. h$.

307. In order to see how remarkably well the total rainfall up to the damaging rains are represented by this parabola, it is only necessary to compare the value $a_h$ calculated from (19) with the observed values. It will be noticed that, numerically, we have

$$(20) \qquad a_h = 1.65 + 2.50 \, (\log. h)^2$$

which gives in inches,

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ ..................... | 1 | 10 | 25 | 50 | 100 | 200 |
| $a_h$ calculated....... | 1.65 | 4.15 | 6.55 | 8.88 | 11.65 | 14.87 |
| $a_h$ observed......... | 1.57 | 4.31 | 6.38 | 8.88 | 9.17 | 4.87 |
| cor. of observation | +0.08 | —0.16 | +0.17 | 0.00 | ......... | ......... |
| The drop ........... | ......... | ......... | ......... | ......... | 2.48 | 10.00 |

I consider the calculated values, each depending on the entire series of observations, more reliable than the observed, and accordingly determine the correction of the observed values.

308. The deviations are entirely insignificant and evenly balanced. The observed points are prominently marked in the diagram, Fig. 7, where the calculated parabola has been accurately drawn.

It is therefore simply an indisputable fact that notwithstanding the apparently irregular succession of rains, my observations, from 1881 to 1891, at Iowa City, show: *The curve of total rainfall is a common parabola, for the rain intensities as abscissæ.*

309. It is manifestly impossible that such a relation could continue indefinitely upwards; accordingly, we find a point reached between the highest useful and the lowest damaging rain, where the curve "drops" in the form of a *straight line.*

310. For the case in hand we know only two points, and hence have no proof that the line continued is really straight. We shall, how-

ever, see that the entire law is general, and will come across cases (Russia, paragraph 340) where more than two points are determined in the drop; these are found to lie exactly in the same straight line. The same is also shown in the monthly curves (paragraph 439).

311. It will be interesting to calculate by formula (20) the corresponding total rainfall for the metrical scale of intensities (paragraphs 70 and 72).

The following table represents the results of this calculation:

| | | | | | | |
|---|---|---|---|---|---|---|
| Intensity (units) | 0 | 1 | 2 | 3 | 4 | 5 |
| $h$ (centi-inches) | 1.2 | 10 | 20 | 40 | 80 | 160 |
| $a$ calculated (inches) | 1.68 | 4.15 | 5.87 | 8.05 | 10.67 | 13.75 |
| $a$ calculated (centi-meters) | 4.27 | 10.54 | 14.91 | 20.44 | 27.10 | 34.92 |

For metrical units of measure the constants of 306 are $a = 4.19$ and $K = 6.35$ centimeters.

This little table will prove very useful for comparison with corresponding results from observations abroad.

312. In Fig. 7 I have also marked the actually observed values for the individual years, so far as they fell not too close to the parabola. It will be understood that a point marked 3 represents the value for 1883; if marked 6 it means 1886, if 0 it is 1890.

313. The points marking the year of the drought, 1886, have been joined by a line to bring out the characteristic features of the rainfall of that year. It will be seen that the greatest peculiarity, bringing the total rainfall of that year very low, is the remarkably small amount of damaging rains. Of the useful rains, the 0.25 inch rains were exceeding low, while the 0.10 inch and 0.50 inch rains were nearly normal.

Consequently the injurious effects were due to the lack of the 0.25 inch rains, and unseasonable occurrence of the 0.10 inch and 0.50 inch rains.

314. The points marking the years 1889 and 1890 (9 and 0) have also been joined by a full drawn and a dotted line respectively. This brings out the striking fact that the useful rains of 1890 were normal, though the total rainfall of that year was very low—the great deficiency being due to the absence of damaging rains.

For the year 1889 the damaging rains were nearly normal, but the useful rains were very low.

Consequently the rains of 1889 were insufficient, those of 1890 favorable for crops, though the total amount of rain in both years was nearly the same. (Compare paragraphs 32 to 43.)

315. It will also be noted that the highest points are marked 3, 2, 1, 3, and 1, representing the years 1883, 1882, and 1881, or the first years of the decennial period. The lowest points are marked 9, 6, 9, 0, 0, representing the closing years 1889 and 1890 of the decennial period and the year of the drought, 1886. We shall see

other evidence of the fact that the first years in the eighties were wet, while the last years were dry.

316. In the decennial period these contrasts have nearly balanced; but if we were to use the lustrum recommended by the international congresses, the two five year's periods of this decennary would exhibit a great contrast. Most assuredly, the lustrum is of no practical utility in climatology. (Compare paragraph 100.)

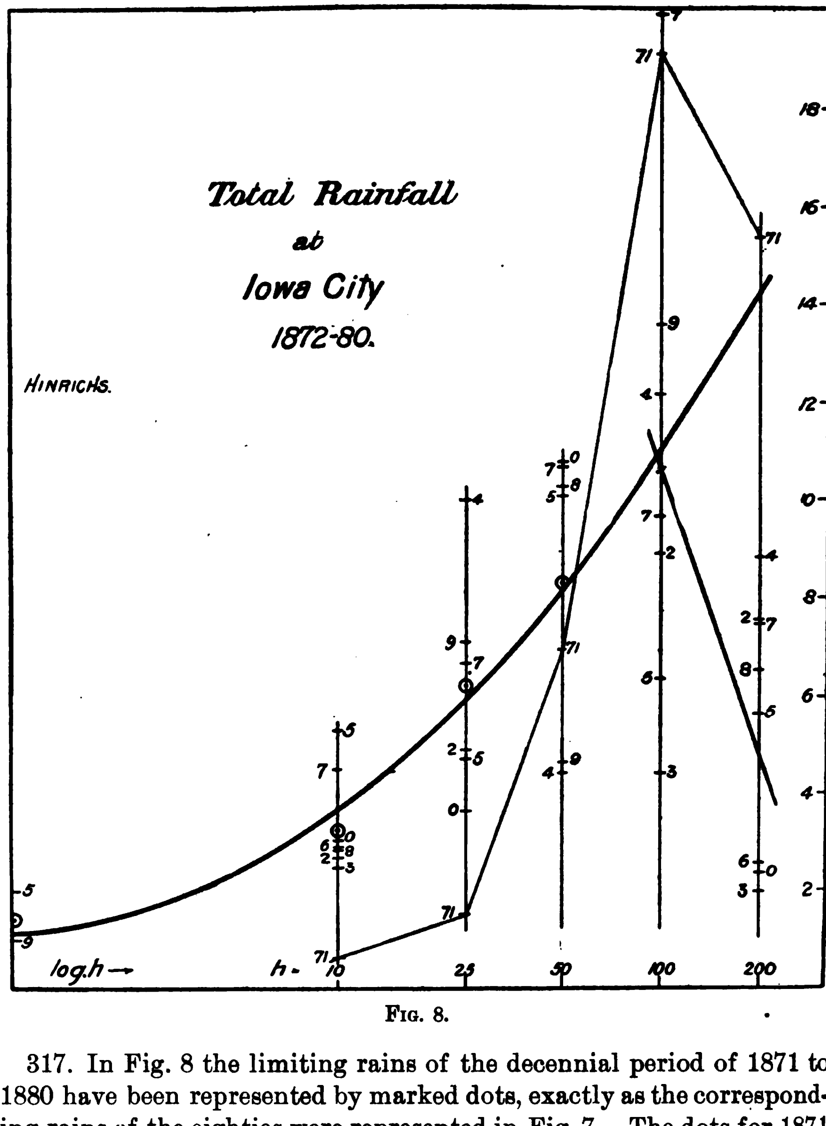


FIG. 8.

317. In Fig. 8 the limiting rains of the decennial period of 1871 to 1880 have been represented by marked dots, exactly as the corresponding rains of the eighties were represented in Fig. 7. The dots for 1871 have been joined by a broken line. It shows exceedingly low values for low intensities and correspondingly high values for high intensities.

318. This exceptional feature is not real. It is due to the method of record used by the observer, Prof. T. S. Parvin, who, under the old Smithsonian rules, measured at the end of showers or series of showers, and not at a stated hour each rain day. For the year 1871 his record shows that he did less of this than formerly; the contrast was so favorable for 1871 that I was in hopes to retain his results in my discussion in order to cover the entire twenty years's period. But so far as the distribution of the total rainfall according to rain intensities is concerned, it is manifestly impossible to do so.

319. Accordingly, I have calculated the mean values for the nine years, 1872 to 1880, in addition to the mean for the decennial period given in the proper tables.

Plotting these nine-year means as before, I find exactly the same parabola, only moved down toward the axis of abscissæ by 0.54 inch; that is, the constant, $a$, of formula (20), paragraph 307, is 1.11, while the parametric value, $K$, is 2.50 inches, as before.

320. The following table gives the comparison between the calculated values and those observed by myself, for the nine years, 1872 to 1880:

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a_h$ calculated | 1.11 | 3.61 | 6.01 | 8.33 | 11.11 | 14.33 |
| $a_h$ observed | 1.42 | 3.28 | 6.28 | 8.40 | 10.64 | 4.79 |
| Correction of obs. | −0.31 | +0.33 | −0.27 | −0.07 | ........ | ........ |
| The drop | ...... | ...... | ...... | ...... | 0.47 | 9.54 |

321. The agreement is equally good as for the eighties, so far as the distribution of errors are concerned, they being alternately negative and positive and nearly equal in amount. But this amount is nearly double that of my series for the eighties. (See paragraph 307.)

322. This is exactly what might have been expected as an expression of the measure of precision of the two series. For the series 1872 to 1880 consists of two parts. The rain gauge was in a public campus, and the readings were not made by one person only until May, 1876. From May, 1876, till 1890, the rain gauge was placed in my garden, and the observations were taken by myself almost always.

323. In order that it may appear more distinctly to what extent the observations of the one year, 1871, modify the mean values, I give the following table:

*Mean total rainfall.*

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| 1871–1880 | 1.34 | 3.25 | 5.80 | 8.45 | 11.49 | 5.85 |
| 1872–1880 | 1.42 | 3.28 | 6.28 | 8.40 | 10.64 | 4.79 |
| 1881–1890 | 1.57 | 4.31 | 6.38 | 8.88 | 9.17 | 4.87 |
| 1872–1890 | 1.50 | 3.80 | 6.33 | 8.65 | 9.80 | 4.83 |

324. It is very apparent that the means of my two series, 1872 to 1880, and 1881 to 1890, are closer than when the first series is made

into a full ten-year period by including Parvin's observations for 1871. The low values are raised, the high values are lowered to a much larger extent than I had anticipated.

325. In other words, my expectation that the deviation of 1871 would not seriously affect the mean of the decennial period has not been realized; the *tenth* of that error is too much to be tolerated in our ten year's means.

326. In conclusion, I will give the values for the twenty year's period, 1871 to 1890, including the year 1871 of Parvin's observations, and also the mean of the nineteen years observed by myself from 1872 to 1890:

*Mean total rainfall.*

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| 1871–1890 | 1.45 | 3.78 | 6.09 | 8.67 | 10.33 | 5.36 |
| 1872–1890 | 1.50 | 3.80 | 6.33 | 8.65 | 9.80 | 4.83 |
| Deviation due to 1871 | —0.05 | —0.02 | —0.24 | +0.02 | +0.53 | +0.53 |

This demonstrates that even in twenty years the one year's different method shows very plainly, lowering the values for low intensities, and raising them very appreciably for high intensities.

327. Since the parameter is the same for the entire series, we have $K = 2.5$ as before, and $a_1$ will be 1.38.

With these values of the constants, $a_1$ and $K$, our formula (20) will give the following calculated values of the mean annual rainfall for each rain intensity:

*Nineteen year's mean values*, 1872–1890.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ calculated | 1.38 | 3.88 | 6.28 | 8.61 | 11.38 | 14.60 |
| $a$ observed | 1.50 | 3.80 | 6.33 | 8.65 | 9.80 | 4.83 |
| Correction of obs. | —0.12 | +0.08 | —0.05 | —0.04 | ........ | ........ |
| The drop | ......... | ......... | ......... | ......... | 1.58 | 9.77 |

328. As might be expected for the longer series, the deviations of the observed from the calculated values are entirely insignificant, being only from 0.04 to 0.08 inch for the useful rains, and only 0.12 inch for insignificant rains.

329. The "drop" marking the discontinuity of the parabolic law is about 1.5 inches at the 1-inch rains and about 10 inches for the 2-inch rains.

In the years commonly called wet years, and marked by the frequency of damaging rains, the parabolic curve is followed higher up than usual. Compare the 1 and 2-inch rains of 1874 on Fig. 8, and those of 1884 on Fig. 7.

330. We have thus discovered a most unexpected simple law which governs the total rainfall as function of rain intensity.

The parabola marking the gradual increase of rain depth with rain intensity (log. $h$) is identical with the form of a jet of water issuing horizontally from a small aperture under the acceleration $2K$ (5

inches for Iowa City), the time, $t$, being counted in intensity, $t = \log. h$, while the position of the aperture is determined by the total rainfall due to the insignificant rains.

## VII.—THE LAW OF TOTAL RAINFALL IS GENERAL.

331. The most remarkable law determining the total rainfall as function of rainfall intensity can not be accidental, for it applies to both ten-year periods of observation at Iowa City and to the entire twenty-year series. Nor can it possibly be a merely local law, though it will no doubt in the constants entering the formula reflect all the varying conditions of rainfall over the globe.

332. If we more closely look at the remarkable graphic representing the two ten years' mean values for Iowa City obtained from our own observations, we will recognize three distinct geometrical elements therein.

333. First, the curve starting on the tangent to the vertex of the parabola; next, the parabolic curve proper, rising upwards, and lastly, the rectilinear "drop," marking the limit of actual rain intensity.

334. These three elements we may expect to find more or less marked everywhere, and under certain climatic conditions one or two of these elements may largely predominate.

Since it can not be expected that I should here give a complete exposition of rainfall analysis, as it will soon develop on the new basis of logarithmic rain intensity, it may suffice to state the general result and give a few numerical data in support of the same.

335. In an extremely continental and dry climate like that of Astrakhan the whole form is scarcely more than the horizontal tangent. For the year in question (1889) we find the amount, $a$, of rainfall at Astrakhan to have been, in millimeters:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 20 | 40 | 80 | 160 |
| $a$ | 33 | 41 | 40 | ... | 34 | ..... |

336. In tropical India it is difficult to assume a higher limit to rain intensity; at any rate I fail to notice any evidence of the final "drop" in the observations of the six stations used. The points determined are marked on Fig. 9 by means of a square drawn around the same. It will be noticed that the three highest points are exactly in a straight line, which is represented by heavy dots in the figure. This line might be taken as the first approximation of the upper portion of the parabolic branch.

337. Since the vertex of the parabola determined by the amount of the insignificant rains lies higher in colder and lower in warmer climates we may probably adopt the base as the tangent to the vertex for India. This means that we overlook the insignificant rains.

338. If we do so, the points for $h$, 10, 25, and 200 are exactly in this parabola, the equation of which we find to be

$$a = 1.85\ (\log.\ h)^2$$

which gives the following results (in inches):

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ calculated | 0 | 1.85 | 3.63 | 5.34 | 7.40 | 9.78 |
| $a$ observed | 1.15 | 1.85 | 3.68 | 6.69 | 8.60 | 10.40 |
| Correction of calc | ...... | 0.00 | +0.05 | +1.35 | +1.20 | +0.62 |

The agreement is quite satisfactory, considering the magnitude of the tropical rains, which are restricted to a few months of the entire year.

339. The six stations in the Russian Empire give very satisfactory results, especially as they allow us to prove the "drop" to be a straight line. The parabola and the "drop" are represented in Fig. 9. The following equations represent these two elements, in millimeters:

$$\text{Parabola } a = 70 + 18\ (\log.\ h)^2$$
$$\text{The drop } a = 114 - 170\ (\log.\ h - 1.600)$$

340. The table here given exhibits the calculated and observed values, always in millimeters:

| $h$ | 1 | 10 | 20 | 40 | 80 | 160 |
|---|---|---|---|---|---|---|
| *Parabola—* | | | | | | |
| $a$ calculated | 70 | 88 | 100.4 | 116.1 | 135.0 | 157.1 |
| $a$ observed | 70 | 90 | 100 | 114 | 63 | 12 |
| Correction of calc | 0 | +2 | —0.4 | —2.1 | ......... | ......... |
| The *drop—* | | | | | | |
| $a$ calculated | ......... | ......... | ......... | 114 | 63 | 12 |
| $a$ observed | ......... | ......... | ......... | 114 | 63 | 12 |
| Correction of calc | ......... | ......... | ......... | 0 | 0 | 0 |

341. It is interesting again to compare the three great regions of Iowa (as type of the Mississippi Valley), India, and Russia. The parabolic branch for Russia is small in comparison with that of Iowa. The parabola for India resembles that of Iowa, but continues much farther; indeed, its ending in a drop has not yet been detected.

342. The constants for these three regions are the following:

| | Inches. | | Millimeters. | | Drop. |
|---|---|---|---|---|---|
| | $a$ | $K$ | $a$ | $K$ | $h$ |
| India, 1881 to 1890 | 0.0 | 1.85 | 0 | 47.2 | 40 |
| Iowa | 1.65 | 2.50 | 43 | 64 | 100 |
| Russia | 2.76 | 0.71 | 70 | 18 | 300? |

343. The "drop" occurs at $h = 40$ centi-inches in Russia, but not till $h = 100$ centi-inches in Iowa, while in India there is no sign of it at $h = 200$, and possibly does not occur till about 300.

344. It will be understood that the higher the rain intensity reaches before the "drop" from the parabolic curve takes place, the longer is

that parabola, and the more extended the range of the regular system of rains increasing in total amount conform to the parabolic law, being proportional to the square of the rain intensity.

345. The rain system of Russia appears small on Fig. 9, while that for India is immense; Iowa stands nearest India, also, in this connection.

346. Only a few other cases need be added to the above. The two stations, Borkum and Keitum, on islands in the North Sea near Holstein, give the constants *a* 66 millimeters and *K* 65 millimeters, with the following result:

| | | | | | | |
|---|---|---|---|---|---|---|
| *h* | 1 | 10 | 20 | 40 | 80 | 160 |
| *a* calculated | 66 | 131 | 176 | 232 | 301 | ..... |
| *a* observed | 66 | 131 | 191 | 230 | 81 | ..... |
| Correction of calculation | 0 | 0 | +15 | —2 | ..... | ..... |

The drop takes place immediately after *h* 40.

347. From central Europe we have taken the three stations, Berlin, Vienna (to represent the continental features), and Aachen, which is under the influence of marine conditions. The parabolic curves start at the same point determined by equal amount of insignificant rains (see Fig. 9), and while for the continental stations the parabola runs parallel to that of Russia, the parabola for Aachen runs nearly parallel to that of the near North Sea.

348. The mean of the two continental stations (Berlin, 1888, and Vienna, 1889), gives *a* 85 millimeters and *k* 20 millimeters, with the following results:

| | | | | | | |
|---|---|---|---|---|---|---|
| *h* | 1 | 10 | 20 | 40 | 80 | 160 |
| *a* calculated | 85 | 105 | 119 | 136 | 157 | ..... |
| *a* observed | 89 | 104 | 198 | 137 | 118 | ..... |
| Correction of calculation | +4 | —1 | ..... | +1 | drop | ..... |

The agreement is complete, except for rains of intensity 20 centiinches, which were largely in excess.

349. For Aachen (also 1888) we find *a* 86 millimeters and *K* 81.5 millimeters, with the following excellent results:

| | | | | | | |
|---|---|---|---|---|---|---|
| *h* | 1 | 10 | 20 | 40 | 80 | 160 |
| *a* calculated | 86 | 168 | 225 | 299 | 379 | ..... |
| *a* observed | 86 | 151 | 224 | 300 | 178 | ..... |
| Correction of calculation | 0 | —17 | —1 | +1 | drop | ..... |

350. The Bergen coast (stations Bergen and Florœ) of Norway exhibits the extreme climatic condition of that noted region of rains even more strikingly in the total rainfall than in the rain frequency. If we take any of the curves of Fig. 9 and raise the same, by the point where the "drop" begins, to an extraordinary height, we shall have left mainly two lines, dropping steeply down from this point. Fig. 9 shows that this is exactly the form which characterizes the "Bergen coast".

351. For Astrakhan we found only the horizontal tangent, for the

Bergen coast we have these two steep lines showing the mountain of rain water precipitated from the air striking the mountains of the coast. These are the two extreme limits included in the general form, shown in perfection by the lines representing Iowa and Russia.

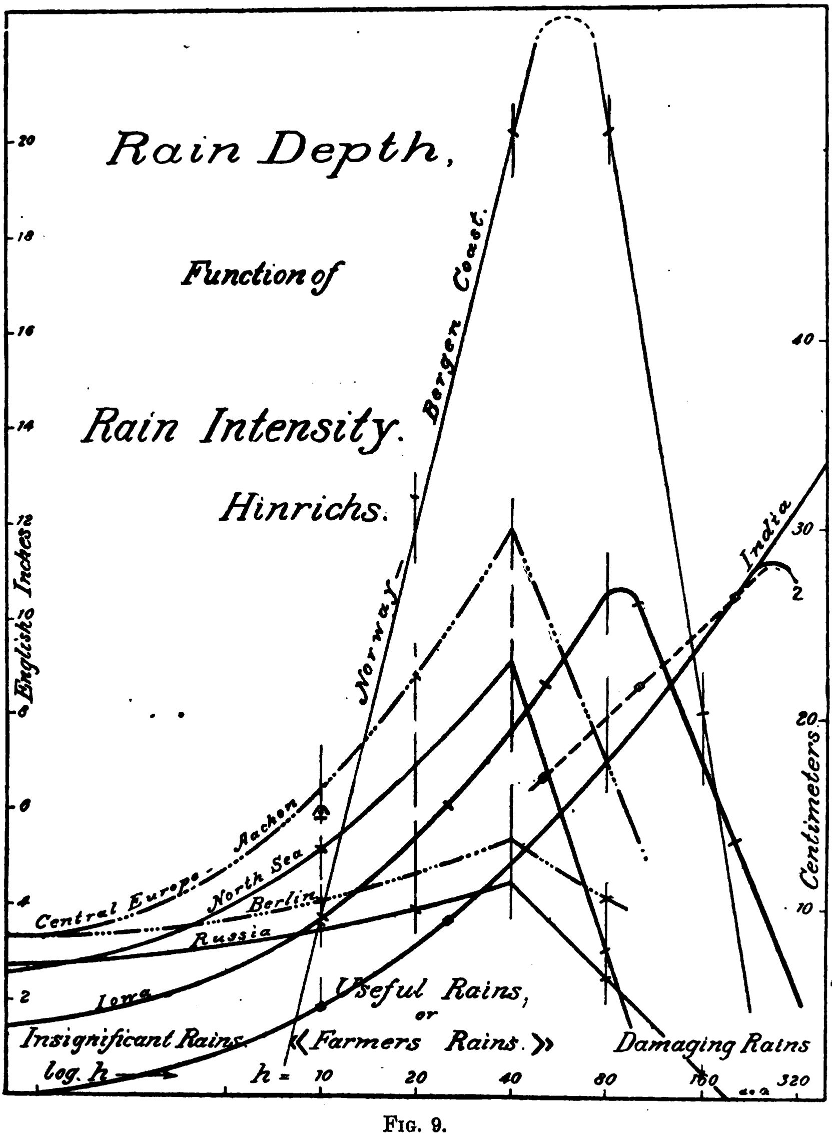


FIG. 9.

352. The rising line for the Bergen coast is represented, in millimeters, by

$$a = 92 + 701\ (\log. h - 1)$$

while the descending line is given by

$$a = 200 + 1036.3\ (2.204 - \log. h)$$

These two lines meet at the point $h$ 60.3 centi-inches and $a$ 639 millimeters.

Calculating the value for $h=20$, we find $a$ 303 against 318 millimeters observed. The observed values are

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 20 | 40 | 80 | 160 |
| $a$ (millimeters) | 54 | 92 | 318 | 514 | 512 | 200 |

353. It is thus completely demonstrated as a matter of fact, that the total rainfall forms a parabola over the rain intensity, which parabola continues until the limit depending on the climatic and seasonal conditions has been attained, when it suddenly "drops" off mainly in the direction of a straight line.

The extreme conditions exemplified by Astrakhan and Bergen, represent limiting forms of this combination of the parabola and the straight line, as shown.

354. To complete the data I here give the table containing the results of observations for the individual stations, the group means of which have been used in the preceding paragraphs. Compare the corresponding table, beginning with paragraph 166:

*Total rainfall in 1889, arranged according to rain intensities.*

| 355. INDIAN EMPIRE.* | English units (in centi-inches). | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 10 | 25 | 50 | 100 | 200 |
| Coast (three stations): | | | | | | |
| Calcutta | 159 | 358 | 768 | 1,929 | 1,022 | 1,380 |
| Madras | 100 | 293 | 323 | 799 | 866 | 1,002 |
| Bombay | 125 | 409 | 541 | 851 | 1,226 | 3,622 |
| Mean | 128 | 353 | 544 | 1,193 | 1,038 | 2,001 |
| Interior (three stations): | | | | | | |
| Nagpur | 127 | 187 | 562 | 710 | 1,809 | 541 |
| Allahabad | 96 | 229 | 684 | 833 | 835 | 1,353 |
| Lucknow | 83 | 194 | 432 | 903 | 1,982 | 1,472 |
| Mean | 102 | 203 | 559 | 815 | 1,542 | 1,122 |
| Mean for coast and interior | 115 | 185 | 368 | 669 | 860 | 1,041 |

| 356. RUSSIAN EMPIRE.† | Metrical units (in centi-inches). | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 10 | 20 | 40 | 80 | 160 |
| Siberia (three stations): | | | | | | |
| Irkutsk | 2 | 50 | 99 | 58 | 41 | 21 |
| Barnaul | 74 | 105 | 124 | 158 | 52 | ....... |
| Tobolsk | 98 | 54 | 84 | 26 | 33 | ....... |
| Mean | 58 | 70 | 102 | 81 | 42 | 7 |
| European Russia (three stations): | | | | | | |
| Kasan | 69 | 90 | 71 | 117 | 27 | ....... |
| Moscow | 91 | 114 | 113 | 159 | 90 | ....... |
| Warsaw | 82 | 127 | 107 | 166 | 136 | 48 |
| Mean | 81 | 110 | 97 | 147 | 84 | 16 |
| Mean for Siberia and European Russia | 70 | 90 | 100 | 114 | 63 | 12 |

* Rain amount in hundredths of an inch.

† Rain amount in millimeters.

*Total rainfall in* 1889, *&c.*—Continued.

| 357. CENTRAL EUROPE.† | Metrical units (in centi-inches). | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 10 | 20 | 40 | 80 | 160 |
| Continental (two stations): | | | | | | |
| Berlin | 93 | 101 | 198 | 139 | 76 | ........ |
| Vienna | 84 | 108 | 199 | 134 | 161 | ........ |
| Mean | 89 | 104 | 198 | 137 | 118 | ........ |
| Aachen | 86 | 151 | 224 | 300 | 178 | ........ |
| 358. NORTH SEA. | | | | | | |
| North Sea (two stations): | | | | | | |
| Keitum | 73 | 132 | 167 | 233 | 69 | ........ |
| Borkum | 60 | 130 | 216 | 227 | 94 | ........ |
| Mean | 66 | 131 | 191 | 230 | 81 | ........ |
| 359. BERGEN COAST. | | | | | | |
| Bergen coast (two stations): | | | | | | |
| Bergen | 53 | 49 | 286 | 504 | 546 | 299 |
| Floroe | 55 | 134 | 351 | 524 | 479 | 102 |
| Mean | 54 | 92 | 318 | 514 | 512 | 200 |

† Rain amount in millimeters.

360. By the formula (2), paragraph 64, log. $h$ really is expressive of a duration or time $t$. Under the influence of an acceleration $2K$, the fall in such a given time will be expressed by

$$a = K\,(\log.\,h)^2$$

which is identical with the law we have discovered to be a fact. As resistance of the air puts a limiting straight line to the parabolic trajectory of a projectile, so does the limit of moisture in the air put a limiting straight line to the parabola marking the total rainfall.

## VIII.—THE LINEAR DEPTH AS INDEPENDENT.

361. The discovery of the important laws regulating the frequency and amount of rainfall in all climes of the globe is in itself a sufficient demonstration of the correctness of the logarithmic rain intensity, which is the foundation of this entire research.

362. Nevertheless, it will be very interesting to also, independently, consider amount and frequency as function of the *linear* rain depth, conformable to the accepted meteorological principles of to-day. We shall, by contrast, show how the general laws remain hidden, thus proving directly that the linear rain depth is *not* the proper independent variable in the mechanics of rainfall.

363. We shall, incidentally, demonstrate the fallacy of some principles that continue to obtrude themselves on the literature of meteorology.

### RAIN FREQUENCY.

364. Taking the mean annual rain frequency, $n$, determined from my best ten years' series of observations at Iowa City, we have the following data of observation:

*Rain frequency*, 1881–1890, *Iowa City.*

| | | | | | | |
|---|---|---|---|---|---|---|
| *h* ...................... | 1 | 10 | 25 | 50 | 100 | 200 |
| *n* ...................... | 44.3 | 27.2 | 18.2 | 13.0 | 7.1 | 1.9 |

365. Plotting these values on the linear heights, *h*, as abscissæ, we obtain the upper hyperbolic curve of Fig. 10, marked "Frequency, *n*." This curve has been very carefully constructed. In the original

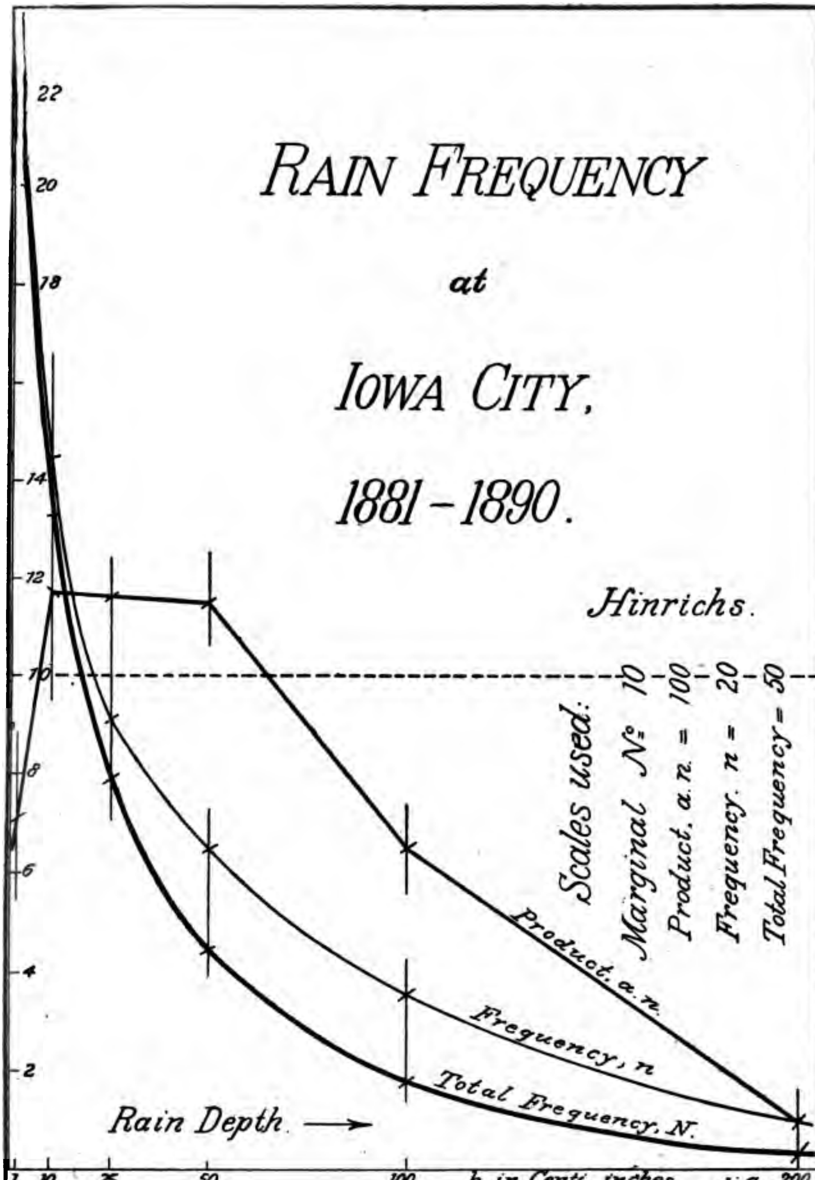


FIG. 10.

drawing the values of $h$ are represented eight times their real length, and the numbers of frequency are exhibited by half an inch as unity. In other words, since the table shows that rains of 0.10 inch, but less than 0.25 inch, occurred one hundred and eighty-two times at Iowa City during the ten years 1881 to 1890, the mean annual frequency of these "tenth-inch rains" is 18.2, and the ordinate of my original drawing of Fig. 10, therefore, was made 9.1 inches in length at the abscissa representing the $h=25$ centi-inches, which abscissa was 2 inches in length.

366. This frequency curve has the appearance of a hyperbola, but it really is not a hyperbola. In fact, it can not be represented by any simple algebraic function. Hence, it follows that the rain frequency can not be expressed as a simple function of the linear rain depth, $h$.

367. How enormously this curve of rain frequency deviates from the simple equilateral hyperbola will be found by determining the product of rain frequency, $n$, into the corresponding rain depth, $h$:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ (centi-inches).. | 1 | 10 | 25 | 50 | 100 | 200 |
| $n$ (frequency)..... | 44.3 | 27.2 | 18.2 | 13.0 | 7.1 | 1.9 |
| Product $hn$...... | 44.3 | 272 | 455 | 650 | 710 | 380 |

368. Even if we disregard the insignificant rains, the product rises gradually from two hundred and seventy-two to seven hundred and ten and thereafter drops rapidly to about half its maximum value.

Consequently, the curve is, in fact, not the remotest approach to a true, equilateral hyperbola.

369. This effectually ought to banish the not uncommon statement that the rainfall amount is inversely proportional to the frequency. Facts do not even remotely approach to such a condition.

370. Taking up the total frequency, $N$, we copy the results of our observations from paragraph 102.

*Total rain frequency*, $N$, *Iowa City*, 1881–1890.

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ (centi-inches)........ | 1 | 10 | 25 | 50 | 100 | 200 |
| $N$........................ | 111.9 | 68.0 | 40.6 | 22.3 | 9.1 | 1.9 |

371. These values are represented in the original drawing of Fig. 10 by taking the 0.20 inch as unit. Hence, the ordinate for 0.25 inch rains is 8.1 inch in length.

It will be noticed that this curve, almost coinciding at $h=1$ with the simple frequency curve $n$, rapidly drops below the latter. It also has the general appearance of a hyperbola.

372. But the curve of total frequency is in no way really a hyperbola. That it is entirely distinct from the equilateral hyperbola is shown by the following table:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$........................ | 1 | 10 | 25 | 50 | 100 | 200 |
| $N$........................ | 111.9 | 68.0 | 40.6 | 22.3 | 9.1 | 1.9 |
| $Nh$...................... | 111.9 | 680 | 1,015 | 1,115 | 910 | 380 |

373. When examined quantitatively, the curve of total frequency fails to show the least similarity to the hyperbola.

There is, accordingly, no simple law connecting the total frequency with the corresponding rain depth, $h$.

374. While engaged in this work, let us also compare the frequency, $n$, with the total rainfall, $a$, corresponding thereto. Suppose we were to construct the curve determined by $a$ as abscissa and $n$ as ordinate, would we find it to be a hyperbola?

375. By calculating the product, $a\,n$, from the observed values of $n$ and $a$, we find, for 1881–1890:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ (centi-inches) ........ | 1 | 10 | 25 | 50 | 100 | 200 |
| $n$ (days) ................ | 44.3 | 27.2 | 18.2 | 13.0 | 7.1 | 1.9 |
| $a$ (inches).......... ..... | 1.57 | 4.31 | 6.38 | 8.88 | 9.17 | 4.87 |
| Product $a\,n$ ............ | 69.6 | 117.3 | 116.1 | 115.4 | 65.1 | 9.2 |

This shows, conclusively, that the curve $a\,n$ is not a hyperbola.

376. It will, however, be noticed that for the useful rains the product $a\,n$ is almost constant, and equal to 116.3—compare the broken line marked "Product, $a\,n$" in Fig 10.

During the ten years from 1881 to 1890, the number, $n$, of the useful rains at Iowa City was inversely proportional to the total amount, $a$, of water which they furnished.

377. For the nine years preceding, the relation was not as close.

*Iowa City*, 1872–1880.

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ (centi-inches)..... | 1 | 10 | 25 | 50 | 100 | 200 |
| $n$ ...................... | 39.9 | 18.4 | 16.1 | 11.2 | 7.1 | 1.7 |
| $a$ (inches)............ | 1.28 | 2.95 | 5.65 | 7.75 | 9.58 | 4.31 |
| Product $a\,.\,n$........ | 61.7 | 92.1 | 114.2 | 109.7 | 74.6 | 9.1 |

378. The product for the useful rains is not nearly as constant, and the mean value, 105.3, is considerably below that for the years 1881–1890.

379. The total amount $a$, evidently is equal to the number of rains, $n$, into the average height, $h^1$, of the same; that is

$$a = h^1\, n$$

accordingly $a\,n$ will be $h^1\, n^2$.

380. We may, therefore, express the above empirical relation in the following manner:

The square of the frequency, $n$, of the useful rains is nearly inversely proportional to the mean of the daily rain depth, $h^1$.

381. Now, the mean rain depth for the useful rains is 0.16, 0.35, 0.68 inch, or nearly as 1 : 2 : 4, that is, as $2^0 : 2^1 : 2^2$. Consequently, the squares of the frequency of these useful rains will be inversely as these squares given. Hence, the frequency of the useful rains approaches the inverse ratio of 1 : 2 : 3, or directly as 6 : 3 : 2. (See paragraph 485.)

382. The practical value of such a rule is very slight; in compari-

son with the general laws of frequency it is nothing. I deemed it appropriate to state this relation simply because it singles out the useful rains from the insignificant and damaging rains.

383. The main positive outcome of this investigation is the direct verification of our principle that the linear rain depth is not the proper independent variable in rainfall investigations.

## RAINFALL AMOUNT.

384. In Fig. 11, I have plotted the rainfall amount as function of linear rain depth, *h*. We know that the rainfall amount first increases more or less gradually to a highest point, and thereafter diminishes with increasing values of *h*.

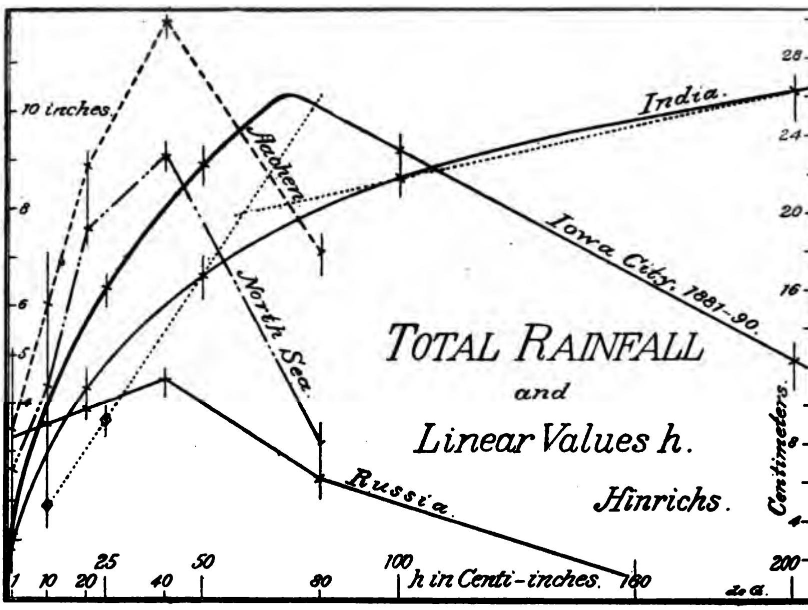


FIG. 11.

385. The points plotted hardly show more than this. The dotted line marked "Aachen" consists of two almost straight lines, the first one, rising, represents the rain amounts from 1 to 20 centi-inches; the second, falling, gives the 40 and 80 centi-inch rains. The case marked North Sea runs in a like manner, but considerably below. Omitting the insignificant rains, the three useful rains of India form a straight line, while the line marking the damaging rains of India continues to rise, but at a much less rapid rate.

The case of Russia resembles the preceding ones, all values being much lower.

The Iowa City values (1881–1890) also mark two such straight lines.

When, however, these lines are investigated more closely they fail to agree, and give too large deviations. Nor can any general relation be given, allowing climatological generalizations.

386. As the first part is rapidly rising we may in some cases place a parabola sufficiently near the points determined, but if we try to do so in all cases we fail. This is most notably the case with the very excellent observations representing Russia.

The parabolæ drawn for Iowa City and India correspond to the general formula

$$(23) \qquad a^2 = K h$$

and show the following results:

387. Iowa City, 1881–1890. $K = 1.65$, hence $a = 1.29 \sqrt{h}$.

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
| $a$ calculated | 1.29 | 4.07 | 6.45 | 9.18 | 12.90 | 18.32 |
| $a$ observed | 1.65 | 4.81 | 6.38 | 8.88 | 9.17 | 4.87 |
| Correction of calc | +0.36 | +0.26 | —0.07 | —0.25 | drop | ....... |

Turning to 317, we find that these deviations are enormous. Expressed in the usual manner, the sum of the squares of these deviations aggregate 2,646, while those in 317 amount to 609 only, or less than one-fourth.

388. The observations for India have also been represented by a parabola in Fig. 2. We find that $K = 0.74$, hence $a = 0.86 \sqrt{h}$, with the following results:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
| $a$ calculated | 0.86 | 2.72 | 4.30 | 6.09 | 8.60 | 12.21 |
| $a$ observed | 1.15 | 1.85 | 3.68 | 6.69 | 8.60 | 10.40 |
| Correction of calc | +0.29 | —0.87 | —0.62 | +0.60 | 0.00 | drop |

The deviations are large for all three of the useful rains, and hence this curve is out of the question.

389. If we were to adopt the linear rain depth as independent variable it would therefore be impossible to represent the rainfall amount by one general law, and in the particular instances the geometrical approximation would be nominal only.

There is no analogy between Russia on the one hand and Iowa and India on the other in the tracings of Fig. 2. Since such analogy exists in fact, these tracings represent no law of nature.

390. In thus having shown that taking the linear rain depth as independent variable it is impossible to find general laws expressive of rainfall frequency and amount, we have added the argument of necessity to the success accomplished by our logarithmic rain intensity scale.

## IX.—MONTHLY FREQUENCY AND AMOUNT.

391. Thus far we have considered the annual values of frequency and amount only.

The year is the principal meteorological period. The earth, completing one revolution in that time, all meteorological conditions will have passed through a corresponding cycle.

392. In this, and in all other periods, the calendar day has been taken as our unit. We have also seen that if this unit is discarded it is impossible to properly utilize the observations made. (See paragraphs 318 and 325.)

About the propriety of the use of these two periods of time, the day and the year, there can be no doubt by meteorologists beyond the point of beginning of the same. The decision of the Meteorological Congress in favor of the calendar year has been properly accepted, but the beginning point of the day at midnight is objectionable for the one element we are considering.

393. For rainfall, the beginning of the day cannot be made at midnight without losing the great bulk of all our observations. It is impossible that the multitude of rainfall observers should measure at midnight.

394. Hence, some other hour than midnight has to be admitted as the beginning of the rain day. I have, for many reasons which it is not necessary here to state, for twenty years adopted noon as the beginning of the day for rainfall. My rainfall days are accordingly mean solar days. So they are also in the volumes of my Iowa Weather Report.

395. We will now take up the rainfall values for the meteorological periods shorter than the year. Here we find the season, the month, the decade, and the pentad.

396. The "meteorological seasons" are a legacy from the time when meteorology was nothing but a "subject" of astronomy.

The four meteorological seasons are simply astronomical quarter years, arbitrarily beginning on the first of March, June, September, and December. They are absolutely worthless from every meteorological standpoint. I shall, therefore, not consider them here.

397. Truly meteorological seasons can only be obtained by the careful discussion of the meteorological observations of a given region or climate. I shall give the proper "seasons" for rainfall in Iowa and the Mississippi Valley, as an example, in this chapter.

398. The calendar month has been admitted by our congress. It is not a meteorological period in any sense, but simply results by roughly dividing the yearly period into a "*dozen*" groups of days. The monthly values fix twelve points in the yearly period; this is not enough to get a correct idea of the course of the annual period. The points being, however, fixed in reference to the yearly period, they are of some practical value in a preliminary investigation of that period.

399. The great length of the month allows too large fluctuations, hence the monthly values are of a very low precision.

400. The decade or ten-day period is sufficiently long not to be too strongly affected by a single ordinary cycle* in the weather and short enough to give a sufficiently large number (thirty-six) of fixed points for the determination of the yearly period. It is also convenient for calculation, most of the decades being truly ten-day periods, only the third decade of the month varying from eight or nine days in February to eleven days in the months of thirty-one days.

401. I have, therefore, steadily made use of the decade as the principal period of the year. In organizing the *first* State Weather Service of America I had the observer's reports mailed at the close of each decade. I have ever continued to use the decade as the proper time sub-unit of the year.

402. In the next chapter the decade results will be considered. Here I shall give the monthly values (calculated from the decade values) simply because the yearly values have already been given, and it will be proper to gradually pass from the longer to the shorter periods.

## MONTHLY TOTAL RAIN FREQUENCY.

403. The mean total rain frequency for each month of the year is, according to my own observations at Iowa City for the ten-year period, 1881 to 1890, summarized in the following table:

*Mean monthly total rain frequency, Iowa City,* 1881–'90.

| | *h* | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 10 | 25 | 50 | 100 | 200 |
| January | 8.9 | 4.9 | 2.0 | 1.0 | 0.2 | 0.0 |
| February | 8.8 | 4.5 | 2.1 | 1.0 | 0.3 | 0.0 |
| March | 8.5 | 5.7 | 2.4 | 0.9 | 0.3 | 0.0 |
| April | 9.6 | 5.5 | 3.4 | 1.3 | 0.3 | 0.1 |
| May | 11.7 | 8.5 | 5.1 | 2.7 | 1.5 | 0.2 |
| June | 12.0 | 9.0 | 6.0 | 4.0 | 1.4 | 0.4 |
| July | 8.6 | 5.2 | 4.0 | 2.5 | 1.7 | 0.4 |
| August | 7.5 | 4.3 | 3.0 | 1.6 | 0.7 | 0.3 |
| September | 9.8 | 5.9 | 3.6 | 2.5 | 0.8 | 0.1 |
| October | 9.2 | 5.8 | 4.0 | 2.4 | 1.2 | 0.3 |
| November | 6.6 | 3.9 | 2.5 | 1.3 | 0.4 | 0.1 |
| December | 10.7 | 4.8 | 2.5 | 1.1 | 0.3 | 0.0 |

Upon closely inspecting these values we notice, first, a marked contrast between the cold half year, from November to April, and the warm half year, from May to October. We next notice that May and June stand distinctly out in the warm half year, while the other four months show a marked difference between the midsummer and the fall months.

*The pentad being so affected is not a practically useful period; when the minor shades are to be studied we must use these daily means.

404. Accordingly, the total rainfall frequency shows that the rain year at Iowa City (representing the middle of the great Mississippi Valley) is first to be divided into four distinct rain seasons, namely:

I. *The season of winter rains*, from November to April; six months.

II. *The rainy season proper*, comprising the months of May and June, and being the "growing season" of the Valley.

III. *The season of midsummer rains*, comprising the months of July and August, the "ripening season."

IV. *The season of fall rains*, comprising September and October, the "harvest months," and the growing season for winter grains. This is the second and minor rainy season of the year.

405. We will consider these rain seasons of the year in detail, determining the constants for each group of months specified, and then calculating the value of the total rain frequency and determining how much has to be added to or subtracted from this calculated value to obtain the number observed.

406. *The season of winter rains*, extending through the six cold months, from November to April, inclusive, gives *a* 0.866 and *b* 0.170, with the following results:

| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| *n* observed | 8.85 | 4.88 | 2.49 | 1.10 | 0.30 | 0.08 |
| *n* calculated | 7.35 | 4.97 | 2.51 | 1.07 | 0.32 | .06 |
| Correction of calc | +1.50 | —0.09 | —0.02 | +0.03 | —0.02 | —0.03 |

Remembering that the observed value of insignificant rains is always too high, the agreement between calculation and observation is complete.

407. *The rainy season* proper, comprising the months of May and June, gives *a* 1.060 and *b* 0.116, with the following results:

| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| *n* observed | 11.85 | 8.75 | 5.55 | 3.35 | 1.45 | 0.30 |
| *n* calculated | 11.45 | 8.79 | 5.53 | 3.10 | 1.36 | 0.44 |
| Correction of calc | +0.40 | —0.04 | +0.02 | +0.25 | +0.09 | —0.14 |

The deviation for the 0.50 inch rains is a little more than desirable, but we should bear in mind the great variation in this, the more or less impetuous rain season of the valley.

408. *The season of midsummer rains*, comprising the months of July and August, give me *a* 0.825 and *b* 0.104, from which the following values were obtained:

| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| *n* observed | 8.05 | 4.75 | 3.50 | 2.05 | 1.20 | 0.35 |
| *n* calculated | 6.68 | 5.52 | 3.47 | 2.06 | 0.98 | 0.46 |
| Correction of calc | +1.87 | —0.77 | +0.03 | —0.01 | +0.22 | —0.11 |

Here the 0.10 inch rains observed were, no doubt, rather low. Probably many rains were near the limit of 0.10 inch.

409. *The season of fall rains*, representing the second or minor rainy season of the Mississippi Valley, so important for fall plowing and for the growth of winter grain, gave me, for the months of September and October, *a* 0.881 and *b* 0.111.

| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| *n* observed | 9.50 | 5.85 | 3.80 | 2.45 | 1.00 | 0.20 |
| *n* calculated | 7.60 | 5.89 | 3.78 | 2.17 | 0.98 | 0.34 |
| Correction of calc | +1.90 | —0.04 | +0.02 | +0.28 | +0.02 | —0.14 |

Here the 0.50 inch rains observed are again in light excess, exactly as in the rainy season.

410. The six winter months may be divided into early, middle, and late winter months. Here it may suffice to consider the four months, January to April, and the early winter months separately.

411. The four months, January to April, give *a* 0.885 and *b* 0.175:

| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| *n* observed | 8.95 | 5.15 | 2.48 | 1.05 | 0.28 | 0.02 |
| *n* calculated | 7.69 | 5.13 | 2.54 | 1.06 | 0.31 | 0.06 |
| Correction of calc | +1.24 | +0.02 | —0.06 | —0.01 | —0.03 | —0.04 |

412. The two early winter months of November and December, combined, give *a* 0.791 and *b* 0.145, also

| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| *n* observed | 8.65 | 4.35 | 2.50 | 1.20 | 0.35 | 0.05 |
| *n* calculated | 6.18 | 4.42 | 2.47 | 1.20 | 0.42 | 0.01 |
| Correction of calc | +2.47 | —0.07 | +0.03 | 0.00 | —0.07 | +0.04 |

The unusually large excess of the insignificant rains is due to their great number in December, giving, therefore, an extraordinary chance to "call" the amount of a sprinkle or a little snow 0.01 inch, which is our lowest limit of record.

413. That even a single month will give perfectly satisfactory results by this reduction, according to our law and method, may be shown in the case of these two winter months. This will also bring out the slight but characteristic difference between the two months.

414. The mean values observed for November give *a* 0.811 and *b* 0.151.

| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| *n* observed | 6.6 | 3.9 | 2.5 | 1.3 | 0.4 | 0.1 |
| *n* calculated | 6.47 | 4.57 | 2.50 | 1.17 | 0.40 | 0.09 |
| Correction of calc | +0.13 | —0.60 | 0.00 | +0.13 | 0.00 | +0.01 |

The observed value for the 0.10 inch rains is too low, nor do the insignificant rains show the usual excess.

415. The mean values observed for the rainiest winter month, December, gives *a* 0.850 and *b* 0.164.

| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| *n* observed | 10.7 | 4.8 | 2.5 | 1.1 | 0.3 | 0.0 |
| *n* calculated | 7.08 | 4.85 | 2.52 | 1.11 | 0.35 | 0.07 |
| Correction calc | +3.62 | —0.05 | —0.02 | —0.01 | —0.05 | —0.07 |

The excessive number of insignificant rains recorded for December was already accounted for above. (See paragraph 412).

416. It will prove interesting to unite the values of the logarithmic constants and of the calculated total frequencies for 1, 10, and 50 centi-inches in a table, precisely as we have done in paragraph 214.

*Table of monthly rain constants—Calculated values.*

| Months. | Logarithms. | | Frequencies. | | |
|---|---|---|---|---|---|
| | $a$ | $b$ | $n_0$ | $n_1$ | $n_3$ |
| January to April | 0.885 | 0.175 | 7.69 | 5.13 | 1.06 |
| May and June | 1.060 | 0.116 | 11.45 | 8.79 | 3.10 |
| July and August | 0.825 | 0.104 | 6.68 | 5.52 | 2.06 |
| September and October | 0.881 | 0.111 | 7.60 | 5.89 | 3.78 |
| November and December | 0.791 | 0.145 | 6.18 | 4.42 | 1.20 |
| November to April | 0.866 | 0.170 | 7.35 | 4.97 | 1.07 |

Examine the curves of Fig. 12, which compare with Figs. 1 and 3.

417. The value of $a$ is greatest for the rainy season of May and June, and least for the early winter months of November and December. The same fact is represented in the corresponding values of the frequency of 0.10 inch rains ($n_1$), which are most numerous (8.8) in May and June, and least so (4.4) in November and December.

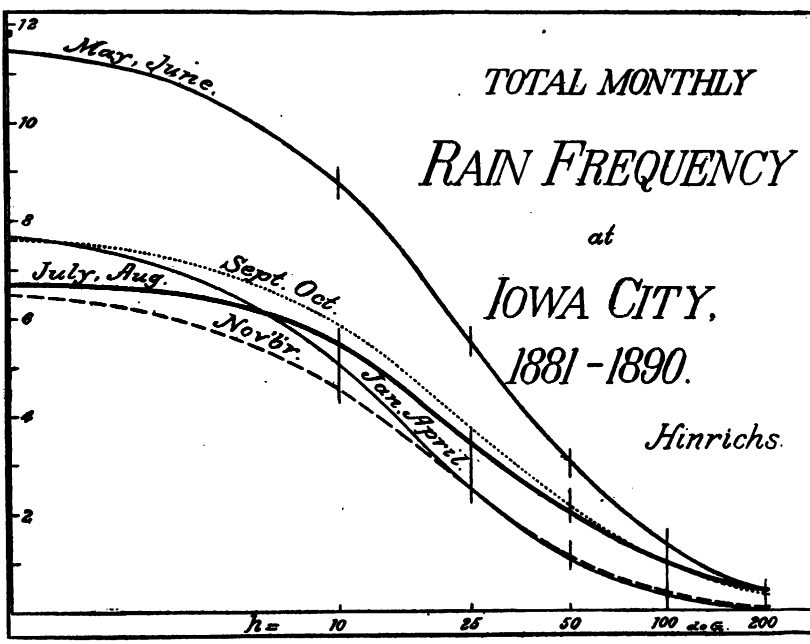


FIG. 12.

418. There is a secondary minimum in midsummer (July, August, 5.5), and a secondary maximum in fall (September, October, 5.9).

419. The rate of decrease ($b$) is greatest in the four winter months, (January to April), and least in midsummer (July, August), dropping from 0.175 to 0.104.

420. It is exceedingly interesting to notice that the value of this decrement $b$ shows but a *single* period in the year, dropping rapidly from the late winter maximum to the midsummer minimum, from which it very gradually rises to the next winter's maximum. We have not time now to enter upon this important topic more fully.

421. The value of this logarithmic decrement $b$ is directly comparable to the values of the same constant given for the mean annual value of different countries in paragraph 214.

We thus see that the low midsummer value $b = 0.104$, is only 0.018 above that of India for the year. Our midsummer rains are like those of India, the high intensities remaining in high percentage.

The midwinter decrement 0.175 is only 0.048 below that of Russia. High intensity rains in our winter are scarce, as are the corresponding rains in Russia.

422. The dual resemblance of the Iowa rainfall both to that of Siberia and India is now more fully understood. Our winter rains resemble the rains of Siberia, and midsummer rains correspond most nearly to those of India, in the distribution of high and low intensity rains.

423. If we desire directly to compare the monthly constant, $a$, with the yearly constant given in paragraph 214, we must, of course, add the value of log. $12 = 1.08$ to the monthly values.

424. We thus find our May and June rains giving the yearly value of $a = 2.14$ exceeding that of central Europe and equaling that of Holstein. If the rain frequency of May and June were to prevail throughout our year we would have a greater number of rain days than central Europe, and as many as Holstein.

425. If the November and December frequency were to continue the entire year the annual value of $a$ would be 1.87, which is only 0.07 below that of Russia. Accordingly our rain frequency would thus fall below that of continental Russia and Siberia.

## MONTHLY RAIN PROBABILITY.

426. From the values of total rain frequency we obtain, in the usual manner, the simple rain frequency giving the mean monthly number of rain days for each rain intensity.

427. The values so obtained are the monthly rain probabilities, and have been represented graphically in Fig. 13. In the original drawing of this photo-engraving the scale of the base is the same as in Fig. 12, but the vertical scale has been doubled.

428. It will be seen that the maximum probability for winter rains

occurs at the low intensity of 0.1 inch, while the summer rains have their maximum at about 0.15 inch.

429. Comparing the curves of Fig. 13 to the corresponding probability curves of Fig. 6 we again recognize that the Iowa winter rains represent Siberian features, while the summer rains correspond to India, not merely in reaching their greatest probability at a higher rain intensity but also in exhibiting a higher degree of symmetry in their distribution.

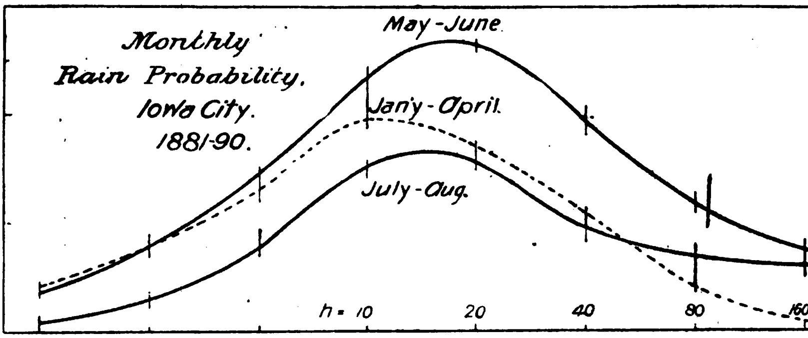


FIG. 13.

430. In concluding this preliminary solution of the long-sought-for probability curve of rain frequency as function of rain intensity, I may be allowed to refer once more to the very meritorious work of Hugo Meyer. This author asserts (p. 134) that the series of rain observations are not sufficiently long to determine merely the maximum values of these probability curves, or the "Scheitelwerth." Even the longest series are declared to be insufficient.

431. Having determined not merely the "Scheitelwerth" but the entire curve of probability for the yearly period of different stations from the observations of a single year, and for the monthly curve from the observations of only ten years, it will, I trust, be admitted that the distinguished author was mistaken when he declared even our longest series, extending over one hundred and fifty years, to be insufficient.

432. I refer to this point, not in a critical spirit, but simply in order to emphasize the fact that it is a question of method of work in this discussion and not one of the massing of further observations.

433. The current scientific conviction, largely due to an unwarranted recrudescence of Baconian empiricism, is detrimental to the progress of every branch of science. No decided step forward into the realms of the unknown has ever been made in that manner. New provinces of truly scientific knowledge have never been conquered by such tactics.

434. Being most uncomfortably convinced by my experience on the

farm and in the garden of the error of our approved meteorological summary of rainfall, I felt the necessity of beginning that earnest study of the mechanism of rainfall briefly sketched in the first chapter. Obtaining in this way the true rain intensity scale, my work was completed in the unerring paths of quantitative induction, the foundation of which was laid by Galileo, the father of modern science.*

435. It was not an additional century of observations of rainfall that was needed; the result of one single year's observation was sufficient to discover the hidden law when the search was made in the light of a full understanding of the rainfall intensity, as measured by the logarithm of the rain depth and not by the linear height of the rainfall.

### MONTHLY TOTAL RAINFALL.

436. Collecting the results of my observations for the ten years from 1881 to 1890, at Iowa City, in rainfall amounts by months and intensities, the following table is obtained:

*Mean total monthly rainfall in hundredths of an inch at Iowa City, Iowa, from* 1881 *to* 1890.

| | *h* | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 10 | 25 | 50 | 100 | 200 |
| January | 14.8 | 43.7 | 32.2 | 59.8 | 21.5 | 0.0 |
| February | 14.4 | 37.4 | 34.5 | 48.8 | 33.0 | 0.0 |
| March | 8.4 | 50.2 | 51.2 | 36.3 | 38.4 | 0.0 |
| April | 13.3 | 31.5 | 73.0 | 65.5 | 23.6 | 29.0 |
| May | 13.2 | 52.3 | 84.4 | 67.4 | 171.4 | 42.3 |
| June | 10.6 | 51.7 | 79.8 | 178.0 | 127.7 | 96.6 |
| July | 12.6 | 21.3 | 56.3 | 51.1 | 175.0 | 100.4 |
| August | 11.8 | 17.9 | 52.0 | 72.3 | 59.0 | 77.7 |
| September | 13.0 | 36.5 | 37.4 | 119.6 | 92.2 | 24.5 |
| October | 11.3 | 30.4 | 54.0 | 75.1 | 109.6 | 89.4 |
| November | 9.0 | 25.2 | 38.7 | 61.4 | 37.3 | 27.1 |
| December | 24.4 | 30.4 | 44.1 | 51.3 | 28.1 | 0.0 |

437. A glance at this table will show that the rain seasons recognized for frequency also appear in this table of total amount. It will, therefore, be sufficient to test our general law of total rainfall on the mean for each of these seasons, though it would be desirable to discuss each month by itself.

438. The four winter months, January to April, give $a_1$ 33.5 and $k$ 7 hundredths of an inch, with the following results calculated by the well-known formula

$$(19, 20) \qquad a_h = a_1 + k\,(\log.\,h)^2$$

| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| *a* observed | 12.7 | 40.7 | 47.7 | 53.1 | 29.6 | 7.2 |
| *a* calculated | 33.5 | 40.5 | 47.5 | 53.5 | ........ | ........ |
| Correction of calc | —20.8 | +0.2 | +0.2 | —0.4 | ........ | ........ |
| The drop | ........ | ........ | ........ | 0.4 | 23.5 | 22.4 |

* Hinrichs' Method of Quantitative Induction in Physical Science. Davenport and Leipzig, 1872.

439. It will be noticed that the useful rains show no appreciable difference between calculation and observation. It is also apparent that "the drop" forms a straight line, since the two consecutive differences are practically identical, differing by .004 of an inch only from the mean, 23.0 hundredths. (See dotted line, Fig. 14.) We have here an exact repetition of the case of Russia, Fig. 9; compare also 310 and 339.

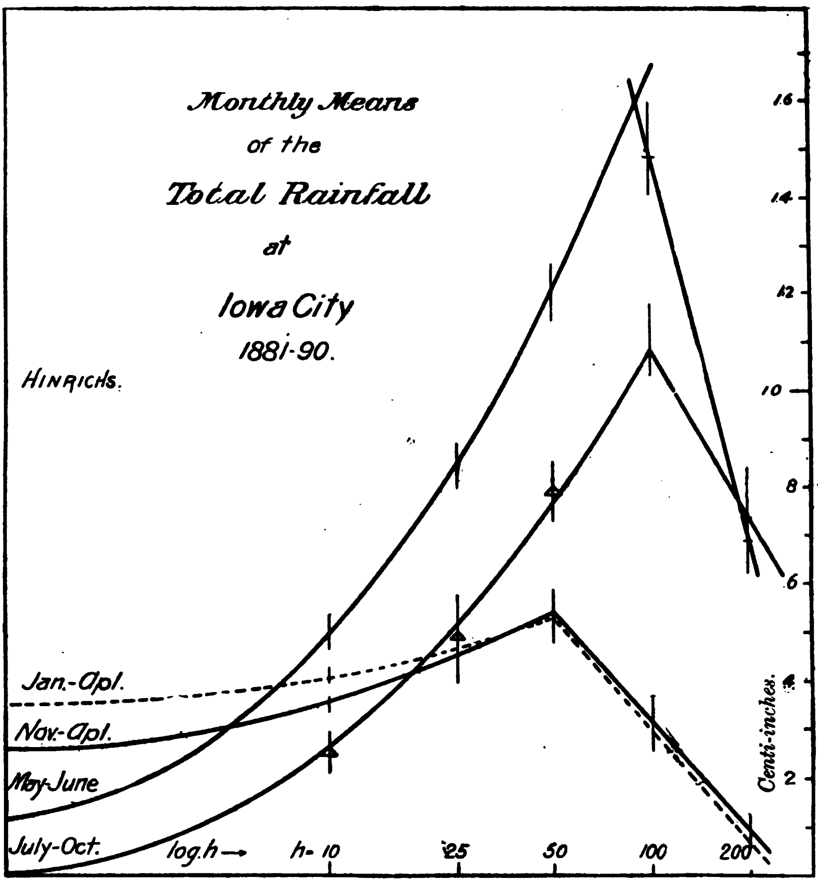


FIG. 14.

440. For the principal rain season of the year, comprising the months of May and June, we find $a_1$ 12 and $k$ 38 giving the following comparison between observation and calculation:

| $h$ ........................ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed ............. | 11.9 | 52.0 | 82.1 | 122.9 | 149.5 | 69.4 |
| $a$ calculated ........... | 12.0 | 50.0 | 86.0 | 122.0 | 164.0 | ........ |
| Correction of calc..... | —0.1 | +2.0 | —3.9 | +0.9 | drop | ........ |

The agreement is as close as can be expected for the season of great

rainfall. At the 1 inch rains the "drop" had just begun, amounting to 14.5 hundredths of an inch.

441. The midsummer rains of July and August give for the mean monthly total rainfall $a_1$ 12 and $k$ 20 hundredths of an inch.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed | 12.2 | 19.6 | 54.1 | 61.7 | 117.0 | 89.0 |
| $a$ calculated | 12.0 | 32.0 | 51.0 | 70.0 | 92.0 | 118.0 |
| Correction of calc. | +0.2 | −12.4 | +3.1 | −8.3 | +25.0 | −29.0 |

Here we have larger deviations than usual, due to the enormous variations of heavy midsummer rains from year to year. The deviations are of alternate sign, however, thus showing that a twenty years' period would bring them to the usually low level.

442. The fall rains of September and October give $a_1$ 12.5 and $k$ 21 hundredths of an inch.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed | 12.2 | 33.5 | 45.7 | 97.4 | 100.9 | 57.0 |
| $a$ calculated | 12.5 | 33.5 | 53.7 | 73.2 | 96.5 | 123.6 |
| Correction of calc. | −0.3 | 0.0 | −8.0 | +24.2 | +4.4 | drop |

The agreement is absolute for $h$ 1, 10, and 100, but the amount of the 0.25 inch rains was markedly low, while the 0.50 inch rains were high. This, no doubt, would disappear in a twenty years' series, and is due to the great variation in the September rains.

443. That such is the case may be readily seen by discussing the October values separately.

The mean total rains of October at Iowa City from 1881 to 1891 give $a_1$ 11 and $k$ 22 hundredths of an inch, with the following admirable results:

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed | 11.3 | 30.4 | 54.0 | 75.1 | 109.6 | 89.4 |
| $a$ calculated | 11.0 | 33.0 | 54.0 | 75.0 | 99.0 | ........ |
| Correction of calc. | +0.3 | −2.6 | 0.0 | +0.1 | +10.6 | drop |

444. Combining the four months from July to October we find the mean monthly total rainfall well represented by the constants $a_1$ 0 and $k$ 27 hundredths of an inch, as shown by the following table of results:

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed | 12.2 | 26.5 | 49.9 | 79.5 | 109.0 | 73.0 |
| $a$ calculated | 0.0 | 27.0 | 52.9 | 78.0 | 108.0 | 142.8 |
| Correction of calc. | +12.2 | −0.5 | −3.0 | +1.5 | +1.0 | −69.8 drop |

445. The tendency in the hot months, accordingly, is to obliterate the insignificant rains, so that the parabola will be tangent to the axis of abscissæ. The agreement then continues far upwards, including the 1 inch rains.

It will be recollected that the yearly rainfall of India exhibits exactly the same features. (See paragraphs 337 and 338.)

446. The early winter months, November and December, give very excellent results for $a_1$ 17 and $k$ 12.5 hundredths of an inch.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed | 16.7 | 27.8 | 41.4 | 56.3 | 32.7 | 18.6 |
| $a$ calculated | 17.0 | 29.5 | 41.5 | 53.2 | 67.0 | 88.3 |
| Correction of calc. | —0.3 | —1.7 | —0.1 | +3.1 | —35.7 | —69.7 |
| | | | | | drop | |

It will be noticed that the drop is again a straight line, being 35.7 and 34.0 for equal intervals on the rain intensities.

447. Taking the six winter months from November to April under one discussion, we find $a_1$ 26 and $k$ 10 hundredths of an inch.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed | 14.0 | 36.4 | 45.6 | 54.2 | 30.6 | 9.3 |
| $a$ calculated | 26.0 | 36.0 | 45.6 | 54.9 | 66.0 | 78.9 |
| Correction calc | —12.0 | +0.4 | 0.0 | —0.7 | —35.4 | —69.6 |
| Drop | ...... | ...... | ...... | ...... | 35.4 | 34.2 |

The mean drop is 34.8, from which the observed values differ by only 0.006 inch; hence, "the drop" forms almost a straight line. (See Fig. 14.)

448. Comparing the curves of Fig. 14, representing the mean monthly total rainfall at Iowa City, from 1881 to 1890, with those of Fig. 9, representing the annual total rainfall in different climates for 1889, it will be noticed again that the winter rainfall in the Mississippi Valley (Iowa) is represented by a curve almost identical with that of Russia, while the midsummer curve (July and August) of Iowa corresponds to that of India. It will also be noticed that the curve for our rainy months of May and June, reaching high above the other monthly curves, reminds us of the lines representing the Bergen coast in Fig. 9, at least sufficiently so to confirm the analysis of the Bergen coast curve given in paragraph 350.

449. In conclusion, we may safely assert that our general law, expressing the total rainfall as function of rain intensity, has been found to represent the monthly mean values as well as the yearly values of observation.

450. It is not deemed necessary to take up my first ten years' series of observation, nor will it be required to demonstrate the law for mean monthly values from other regions of the globe. The demonstration given will, I trust, be considered perfectly sufficient.

## X.—DECADE FREQUENCY AND AMOUNT.

451. The most competent authority has declared even our longest series of observations insufficient for the determination of merely the monthly maximal value of the rain probability as function of intensity (see paragraph 430). How many additional centuries of observation would be required to determine the "Scheitelwerth" by decades?

452. My method of discussion has not merely determined the "Scheitelwerth," but the entire rain probability curve of each month by the use of ten years' series of observations, while the rain probability curve for the year was determined by a single year's observations.

453. While this achievement is more than sufficient evidence of the importance of the new laws, and fully demonstrates that our fundamental principle of the logarithmic rain intensity scale is true to nature, it seemed advisable to apply the laws to the individual decades even.

454. The complete success of this endeavor gives an additional and most weighty confirmation of the laws and principles here obtained by quantitative induction, and at the same time strengthens and confirms my estimates of the climatological value of the ten-day period, the decade (paragraph 400).

455. This entire research having shown the primary importance of rain frequency, we must expect the most perfect results in the examination of the decade frequency, while the decade amount of rainfall will exhibit larger fluctuations.

## DECADE TOTAL RAINFALL.

456. It will suffice to take up a few striking cases, and show that our familiar formula

$$(19, 20) \qquad a_h = a_1 + k\ (\log.\ h)^2$$

represents the facts observed. It need scarcely be stated, that the constants are determined by our graphical method, and involve simply the drawing of the straight line represented by the above formula. (See paragraphs 118 and 119.)

457. The first decade of May generally ushers in our rainy season. Both in frequency and amount it is normally surpassed only by the first decade of June.

It will, therefore, be advisable to consider these two decades, the most marked of our rainy season. We will also consider the second decade of August, the most typical of our midsummer rains.

458. My ten years' observation from 1881 to 1890, at Iowa City, give $a_1$ 10.7 and $k$ 10.8 hundredths of an inch for the first decade of May, with the following results:

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed | 3.0 | 21.5 | 32.9 | 40.7 | 58.9 | ........ |
| $a$ calculated | 10.7 | 21.5 | 31.8 | 41.9 | 58.9 | 67.7 |
| Correction of calc... | —6.3 | 0.0 | +1.1 | —1.2 | 0.0 | ........ |

The agreement is most complete, barring the slight deficiency in the insignificant rains observed.

459. The first decade of June, 1881–'90, gives $a_1$ 6.6 and $k$ 20.6.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed | 2.9 | 27.2 | 38.5 | 66.0 | 33.3 | 45.9 |
| $a$ calculated | 6.6 | 27.2 | 46.9 | 66.1 | 89.0 | 115.5 |
| Correction calc | —3.7 | 0.0 | —8.4 | —0.1 | ........ | ......... |
| Drop | ........ | ...... | ........ | ........ | 55.7 | 70.6 |

Excepting that the 0.25 inch rains were slightly deficient, the agreement is excellent. It will also be noticed that the value $k$ for the June decade is twice as large as for the May decade, representing the great increase in depth of the June rains.

460. Our midsummer rains are represented by the second decade of August, giving $a_1$ 4.2 and $k$ 4.0.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $a$ observed | 4.4 | 8.2 | 12.1 | 13.9 | 16.5 | 23.4 |
| $a$ calculated | 4.2 | 8.2 | 12.0 | 15.8 | 20.2 | 25.4 |
| Correction calculated | +0.2 | 0.0 | +0.1 | —1.9 | —3.7 | —2.0 |
| Drop | ........ | ...... | ........ | 1.9 | 1.8 | ........ |

The formula represents the observations perfectly.

461. By placing the constants in a table, we obtain a clearer insight in the most notable climatological contrast between our ordinary "rainy season" and the midsummer "dry spell" than can be obtained by any other means:

| Decade: | $a_1$ | $k$ |
|---|---|---|
| May I | 10.7 | 10.8 |
| June I | 6.6 | 20.6 |
| August II | 4.2 | 4.0 |

The insignificant rains diminish from 11 to 4 hundredths of an inch, while the parametric constant rises to its double from May I to June I, and sinks to the fifth of the latter value in midsummer.

### DECADE TOTAL RAIN FREQUENCY.

462. Also, here, it will be necessary to restrict our attention to the most important of the decades of the rain year. It is out of the question to take up each one of the thirty-six decades of the ten years' series, 1881 to 1890, in this exposition of the fundamental laws of rainfall. I reserve this exposition for another occasion.

463. It will be exceedingly interesting to consider all the decades of our real rainy season. The study of these six decades will reveal a secondary periodicity of great importance, and furnish an excellent example of what can be accomplished in detail work by the new method of discussion.

464. In addition we will take the second decade of August to represent the least frequency in midsummer; the second decade of November to represent the lowest frequency of the entire year, and the third decade of December, giving the highest number of insignificant

rains and showing the steep decline, the Siberian mark, in highest perfection, with the increase of rain intensity.

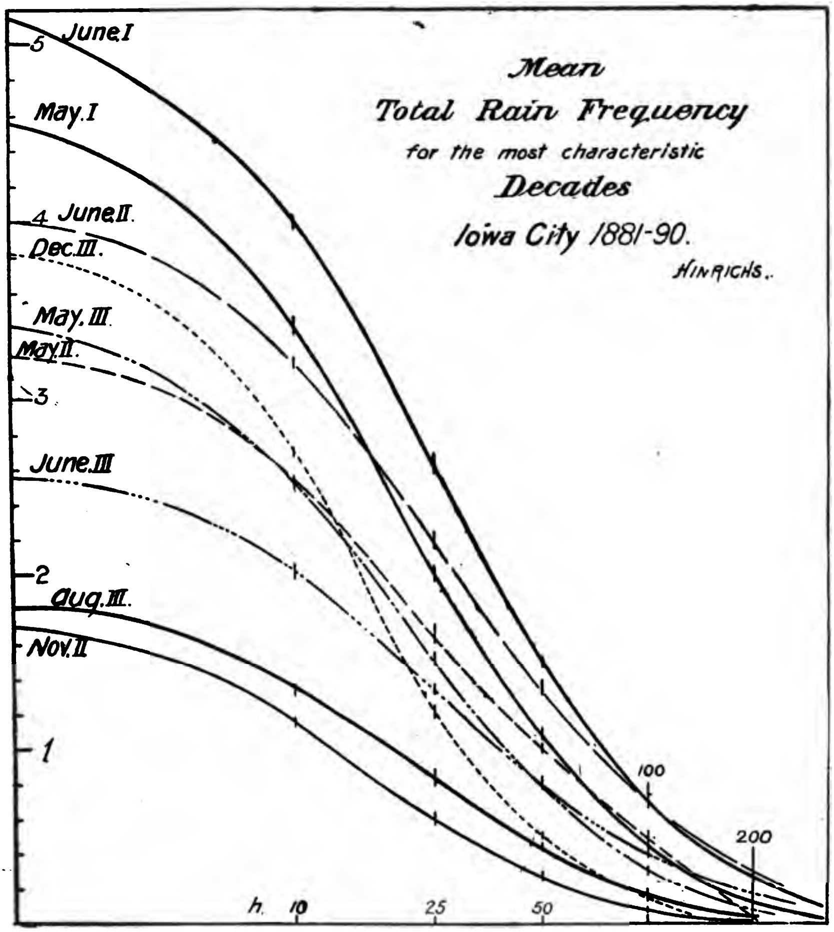


FIG. 15.

465. The first decade of May gives $a$ 0.658 and $b$ 0.126 in formula (6), paragraph 118.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $n$ observed | 4.3 | 3.4 | 2.0 | 1.1 | 0.4 | 0.0 |
| $n$ calculated | 4.55 | 3.40 | 2.06 | 1.10 | 0.45 | 0.13 |
| Correction of calc ... | —0.25 | 0.00 | —0.06 | 0.00 | —0.05 | —0.13 |

466. The second decade of May furnishes, by my graphic method, $a$ 0.510 and $b$ 0.103.

| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $n$ observed | 3.6 | 2.6 | 1.6 | 1.0 | 0.7 | 0.1 |
| $n$ calculated | 3.24 | 2.55 | 1.65 | 1.01 | 0.49 | 0.02 |
| Correction of calc ... | +0.36 | +0.05 | —0.05 | —0.01 | +0.21 | +0.08 |

467. The third decade of May gives $a$ 0.533 and $b$ 0.129; also

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ ........................ | 1 | 10 | 25 | 50 | 100 | 200 |
| $n$ observed............. | 3.8 | 2.5 | 1.5 | 0.6 | 0.4 | 0.1 |
| $n$ calculated........... | 3.41 | 2.54 | 1.51 | 0.79 | 0.32 | 0.09 |
| Correction of calc ... | +0.39 | —0.04 | —0.01 | —0.19 | +0.08 | +0.01 |

468. For the first decade of June I find $a$ 0.711 and $b$ 0.109, giving—

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ ........................ | 1 | 10 | 25 | 50 | 100 | 200 |
| $n$ observed............. | 4.9 | 4.0 | 2.5 | 1.5 | 0.5 | 0.2 |
| $n$ calculated........... | 5.14 | 4.00 | 2.59 | 1.50 | 0.69 | 0.24 |
| Correction of calc ... | —0.24 | 0.00 | —0.09 | 0.00 | —0.19 | —0.04 |

469. The second decade of June give $a$ 0.602 and $b$ 0.095.

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ ........................ | 1 | 10 | 25 | 50 | 100 | 200 |
| $n$ observed............. | 4.0 | 3.0 | 2.2 | 1.6 | 0.7 | 0.1 |
| $n$ calculated........... | 4.00 | 3.21 | 2.19 | 1.37 | 0.70 | 0.28 |
| Correction of calc ... | 0.00 | —0.21 | +0.01 | +0.23 | 0.00 | —0.18 |

470. The third decade of June furnishes $a$ 0.405 and $b$ 0.102.

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ ........................ | 1 | 10 | 25 | 50 | 100 | 2.00 |
| $n$ observed............. | 3.1 | 2.0 | 1.8 | 0.8 | 0.2 | 0.1 |
| $n$ calculated........... | 2.54 | 2.01 | 1.84 | 0.80 | 0.39 | 0.15 |
| Correction of calc ... | +0.56 | —0.01 | —0.04 | 0.00 | —0.19 | —0.05 |

471. Tabulating the characteristic constants for our rain season in the well-known manner (see paragraphs 214 and 416), we obtain the following data, which should be compared to Fig 15.*

From these data I have also constructed the actual rain probability curve for three leading decades in Fig. 16.

*Table of decade rain constants—Calculated values.*

| Decades. | Logarithms. | | Total frequencies. | | |
|---|---|---|---|---|---|
| | $a$ | $b$ | $n_0$ | $n_1$ | $n_3$ |
| May, I........................................ | 0.658 | 0.126 | 4.55 | 3.40 | 1.10 |
| May, II....................................... | 0.510 | 0.103 | 3.24 | 2.55 | 1.01 |
| May, III...................................... | 0.533 | 0.129 | 3.41 | 2.54 | 1.51 |
| June, I....................................... | 0.711 | 0.109 | 5.14 | 4.00 | 1.50 |
| June, II...................................... | 0.602 | 0.095 | 4.00 | 3.21 | 1.37 |
| June, III..................................... | 0.405 | 0.102 | 2.54 | 2.01 | 0.80 |

472. This table shows the periodicity in our rainy season referred to above (paragraph 463). The study of the April rains shows that the first decade of May stands out clearly above April as a rainy decade, having some rain on half its days, over 0.10 inch on one-third its days, and over 0.50 inch on one day.

473. This intensity is not kept up, the second and third decades having only as many measurable rains as the first decade had useful and damaging rains.

* The abscissa of the insignificant rains in the figure is only four-fifths of its true length. The error is not of sufficient consequence to require a redrawing of the figure.

474. By the first of June the grand effort of the year is made, giving us almost tropical rains. It rains on half the number of days, and on four of the ten days the rains exceed 0.10 inch, while on three days in two years over 0.50 inch falls.

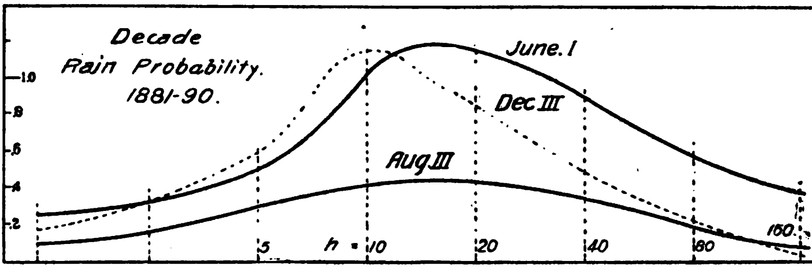


FIG. 16.

475. From this maximum of the entire year's rainfall, marking the first decade of June, the rain frequency very gradually drops, approaching the midsummer level in the last decade of the month.

476. The lowest midsummer rain frequency occurs in the second decade of August, giving *a* 0.255 and *b* 0.124:

| | | | | | | |
|---|---|---|---|---|---|---|
| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
| *n* observed | 2.0 | 1.2 | 0.9 | 0.4 | 0.2 | 0.1 |
| *n* calculated | 1.80 | 1.35 | 0.82 | 0.44 | 0.18 | 0.06 |
| Correction of calc. | +0.20 | −0.15 | +0.08 | −0.04 | +0.02 | +0.04 |

477. The lowest total rain frequency is shown by the second decade of November, giving *a* 0.230 and *b* 0.164:

| | | | | | | |
|---|---|---|---|---|---|---|
| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
| *n* observed | 1.7 | 1.1 | 0.7 | 0.2 | 0.1 | 0.0 |
| *n* calculated | 1.70 | 1.16 | 0.60 | 0.27 | 0.08 | 0.02 |
| Correction of calc. | 0.00 | −0.06 | +0.10 | −0.07 | +0.02 | −0.02 |

478. The "rainiest" winter period is the third decade of December, giving *a* 0.582 and *b* 0.180:

| | | | | | | |
|---|---|---|---|---|---|---|
| *h* | 1 | 10 | 25 | 50 | 100 | 200 |
| *n* observed | 4.7 | 2.5 | 1.2 | 0.5 | 0.1 | 0.0 |
| *n* calculated | 3.82 | 2.52 | 1.23 | 0.50 | 0.14 | 0.002 |
| Correction of calc. | +0.88 | −0.02 | −0.03 | 0.00 | −0.04 | −0.002 |

479. It is difficult to conceive of any closer agreement between observed and calculated values. Nor are we comparing the annual means of the observation of a couple of centuries, but the decade means of only ten years' observation.

This remark applies to nearly all the decades considered with almost equal force.

It is also well understood that the nine decades have not been selected from the entire series of thirty-six to obtain the best agreement between calculation and observation, but solely to exhibit the most characteristic and most important rain periods of the year.

480. I fail to see that anything is lacking in the demonstration of the absolute justness of my logarithmic rain-intensity scale, or in the fundamental laws obtained by quantitative induction and governing the total rain frequency and amount.

### THE MEAN RAINFALL OF ONE DAY.

481. Having determined the rain constants for the year, the month, and the decade, it only remains to determine the mean total amount of rainfall of a single day.

482. The tables of observations give the total amount, $a$, of rainfall for any given intensity, and also the total number, $n$, of rain days of that intensity. Hence, the mean rainfall, $h^1$, of a rain day of the same intensity will be obtained by dividing the total rainfall, $a$, by the number, $n$, of rain days, as expressed below:

$$h^1 \doteq \frac{a}{n}$$

483. The complete ten-year period of my observations at Iowa City, from 1881 to 1890, gives the following results, in hundredths of an inch:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ .............. | 1 | 10 | 25 | 50 | 100 | 200 |
| $a$ amount..... | 1,568 | 4,205 | 6,376 | 8,881 | 9,167 | 4,870 |
| $n$ rain days... | 443 | 272 | 182 | 130 | 71 | 19 |
| $h^1$ mean days | 3.5 | 15.8 | 35.0 | 68.3 | 129.1 | 256.3 |

484. My own observations for the preceding decennial period do not include the year 1871, and since we have seen that it is impossible to utilize Prof. Parvin's observations of 1871 for such purposes (see paragraph 218), we can only use my nine-year observations, from 1872 to 1880. They give, in the same units:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ .............. | 1 | 10 | 25 | 50 | 100 | 200 |
| $a$ amount..... | 1,276 | 2,949 | 5,648 | 7,758 | 9,575 | 4,307 |
| $n$ rain days... | 399 | 184 | 161 | 112 | 71 | 17 |
| $h^1$ mean day.. | 3.2 | 16.0 | 35.1 | 69.2 | 134.9 | 253.3 |

485. Consolidating these two series, we obtain the following table:

*Mean rainfall of one day at Iowa City, in hundredths of an inch.*

| $h$ ...................... | 1 | 10 | 25 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| 1872–'80............. | 3.2 | 16.0 | 35.1 | 69.2 | 134.9 | 253.3 |
| 1881–'90............. | 3.5 | 15.8 | 35.0 | 68.3 | 129.1 | 256.3 |
| 1872–'90............. | 3.3 | 15.9 | 35.0 | 68.7 | 131.9 | 254.8 |

486. The agreement of the two series is truly marvelous; the two series of numbers being practically identical. Even for the damaging rains, the individual series differ from the mean by only two per cent for the full inch rains, and by only half a per cent for the 2-inch rains. For the useful rains the series certainly must be called identical.

487. We are, therefore, in the position to assert, as the most general expression of my entire series of careful rainfall observations extending over the nineteen years, from 1872 to 1890, at Iowa City, that *the mean daily rainfall of a rain day has remained constant.*

This is a new contribution to the series of facts we have discovered, all indicating a most unexpected constancy in the rain phenomenon, which is generally considered the most variable of all meteorological phenomena.

It will be readily understood that this is entirely independent of any possible change in the distribution of the rain intensity throughout the year.

488. Let us see whether we can not determine the mean rainfall of a rain day theoretically. Looking at the curve representing the frequency, $n$, in Fig. 10, we observe that a triangular area marks the decrease of $n$ from each intensity to the next.

If we desire to have the mean height, $h$, it is evidently determined by the center of gravity of this triangle. But the center of gravity is at the distance of one-third from its base. Consequently, the mean height, $h^1$, of a rain day is equal to the height, $h$, at the beginning of the intensity, increased by one-third the increase, $d$, in the scale. That is, we must have

$$(25) \qquad h^1 = h + \frac{d}{3}$$

489. Testing this law by the observed data for the best series (1881–1890), we find that the calculated values correspond very well with the observed ones, as shown in the table here given:

| | | | | | | |
|---|---|---|---|---|---|---|
| $h$ | 1 | 10 | 25 | 50 | 100 | 200 |
| $d$ | 9 | 15 | 25 | 50 | 100 | $x$ |
| $\frac{d}{3}$ | 3 | 5 | 8.3 | 16.6 | 33.3 | $\frac{x}{3}$ |
| $h^1$ calculated | 4.0 | 15.0 | 33.3 | 66.6 | 133.3 | ........ |
| $h^1$ observed | 3.5 | 15.8 | 35.0 | 68.3 | 129.1 | 256.3 |
| Correction for calc | —0.5 | +0.8 | +2.3 | +1.7 | —4.2 | ........ |

490. The upper limit beyond $h = 200$ being unknown, we may approximately calculate that limit from the rule (25) which has just been shown to give results but very slightly below the observed values. If the observed mean rainfall 256.3 is to be equal to the initial value 200 increased by one-third the difference in scale of rain units, this value will be three times 56.3 or 169, making the upper value *de facto* 369 hundredths of an inch for the period of ten years, 1881 to 1890.

Now, during these ten years I observed only three rain days with rainfall in excess of three inches, namely, one in each of the following decades:

Second decade of October, 1881, with 3.32 inches.

First decade of October, 1884, with 3.02 inches.

Third decade of August, 1885, with 3.18 inches.

It will be recognized that none of these rains reach the limit of 3.69 inches calculated, which limit, therefore, is not in conflict with the observations.

### SUMMARY OF OBSERVATIONS.

*Rainfall observations at Iowa City, Iowa.*

| Year. | Total. | | Rain intensity (centi-inches).* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1 | | 10 | | 25 | | 50 | | 100 | | 200 | |
| | *n* | *a* | *n* | *a* | *n* | *a* | *n* | *a* | *n* | *a* | *n* | *a* | *n* | *a* |
| 1871........ | 74 | 4566 | 15 | 61 | 22 | 301 | 5 | 152 | 11 | 698 | 14 | 1916 | 7 | 1539 |
| 1872........ | 86 | 3334 | 32 | 94 | 17 | 267 | 15 | 492 | 12 | 862 | 7 | 892 | 3 | 757 |
| 1873........ | 84 | 2489 | 32 | 108 | 17 | 247 | 17 | 625 | 13 | 878 | 4 | 441 | 1 | 200 |
| 1874........ | 109 | 3975 | 48 | 180 | 16 | 244 | 27 | 1002 | 6 | 443 | 9 | 1223 | 3 | 885 |
| 1875........ | 121 | 3398 | 53 | 201 | 32 | 531 | 14 | 473 | 15 | 1011 | 5 | 637 | 2 | 565 |
| 1876........ | 124 | 4179 | 56 | 154 | 19 | 290 | 20 | 678 | 14 | 909 | 14 | 1993 | 1 | 255 |
| 1877........ | 120 | 4119 | 49 | 189 | 26 | 448 | 19 | 669 | 16 | 1074 | 7 | 968 | 3 | 751 |
| 1878........ | 164 | 3810 | 44 | 147 | 18 | 285 | 16 | 632 | 16 | 1032 | 7 | 1061 | 3 | 655 |
| 1879........ | 97 | 2962 | 37. | 92 | 21 | 334 | 22 | 712 | 6 | 462 | 11 | 1364 | 0 | ....... |
| 1880........ | 97 | 2948 | 46 | 111 | 18 | 303 | 11 | 365 | 14 | 1082 | 7 | 996 | 1 | 239 |
| 1881........ | 122 | 4564 | 49 | 170 | 25 | 392 | 18 | 619 | 20 | 1306 | 5 | 741 | 5 | 1336 |
| 1882........ | 133 | 4006 | 57 | 199 | 26 | 405 | 27 | 1037 | 12 | 857 | 8 | 1008 | 2 | 500 |
| 1883........ | 124 | 3883 | 42 | 146 | 38 | 620 | 20 | 704 | 12 | 852 | 12 | 1461 | 0 | ....... |
| 1884........ | 134 | 3936 | 48 | 182 | 39 | 590 | 24 | 823 | 14 | 917 | 8 | 1122 | 1 | 302 |
| 1885........ | 113 | 4235 | 41 | 119 | 30 | 512 | 15 | 522 | 15 | 1016 | 8 | 1020 | 4 | 1046 |
| 1886........ | 101 | 2464 | 46 | 174 | 28 | 455 | 9 | 302 | 13 | 837 | 3 | 443 | 1 | 260 |
| 1887........ | 104 | 3057 | 44 | 153 | 24 | 369 | 15 | 543 | 12 | 845 | 9 | 1157 | 0 | ....... |
| 1888........ | 101 | 3675 | 42 | 168 | 20 | 315 | 16 | 563 | 12 | 878 | 8 | 1036 | 3 | 715 |
| 1889........ | 90 | 2852 | 39 | 142 | 18 | 270 | 13 | 453 | 9 | 569 | 7 | 814 | 2 | 488 |
| 1890........ | 97 | 2705 | 35 | 115 | 24 | 377 | 25 | 810 | 11 | 804 | 3 | 365 | 1 | 223 |
| **Means.** | | | | | | | | | | | | | | |
| 1871-'80.... | 101.6 | 3580.0 | 41.4 | 133.7 | 20.6 | 325.0 | 16.6 | 580.0 | 12.3 | 845.1 | 8.5 | 1149.1 | 2.4 | 584.6 |
| 1881-'90.... | 111.9 | 3537.7 | 44.3 | 156.8 | 27.2 | 430.5 | 18.2 | 637.6 | 13.0 | 888.1 | 7.1 | 916.7 | 1.9 | 487.0 |
| 1871-'90.... | 106.8 | 3559.0 | 42.8 | 145.0 | 23.4 | 378.0 | 17.4 | 609.0 | 12.6 | 867.0 | 7.8 | 1033.0 | 2.1 | 536.0 |
| 1872-'80.... | ...... | ....... | 44.3 | 141.8 | 20.4 | 327.7 | 17.9 | 627.6 | 12.4 | 839.9 | 7.9 | 1063.9 | 1.9 | 478.6 |

* It will hardly need reiteration that the real intensities are proportional to the logarithms of the centi-inches stated at the head of these columns. The full heading might properly read: Rain intensities corresponding to the rain heights amounting to 1, 10, 25, 50, 100, 200 centi-inches.

*a* = amount in hundredths of an inch.

*n* = number of rain days of each intensity.

1871, observations of Prof. T. S. Parvin. (See paragraph 318.)

1872-1890, observations of Dr. Gustavus Hinrichs.